Electrical Tech Note — 226

Biosystems & Agricultural Engineering Department
Michigan State University

Power Factor

Many types of electrical devices operate on alternating current (ac) and require a magnetic field in order to operate. The most common example is an electric motor. Some ac electrical equipment also contain a component called a capacitor. These components also cause the ac current sine wave to become out of alignment with the voltage ac sine wave as shown in *Figure 226.2* of an electric motor

Alternating current flows in a circuit in a sine wave pattern, flowing in one direction when positive (+) and in the opposite direction when negative (-). For ac power in the U.S. and Canada these plus/minus cycles occur at a rate of 60 every second which is known as 60 Hz power. The voltage causes the current to flow. In the case of alternating current both the voltage and the current occur in the conductors in the form of a sine wave as shown in *Figure 226.1.* For some electrical loads the voltage and current sine waves are in alignment as shown in *Figure 226.1*, and in many circuits the voltage and current sine waves are out of alignment as shown in *Figure 226.2.*

Some ac circuits power resistive loads such as incandescent lamps and resistance type electric heating elements. An electric water heater uses a resistive heating element to heat the water. Space heaters use resistive heating elements. For resistive ac circuits the voltage and current sine waves are basically in alignment as shown in *Figure 226.1.* **Power factor (pf)** is a number that ranges from zero to one (0.0 to 1.0) that indicates how much the voltage and current sine wave are out of alignment.

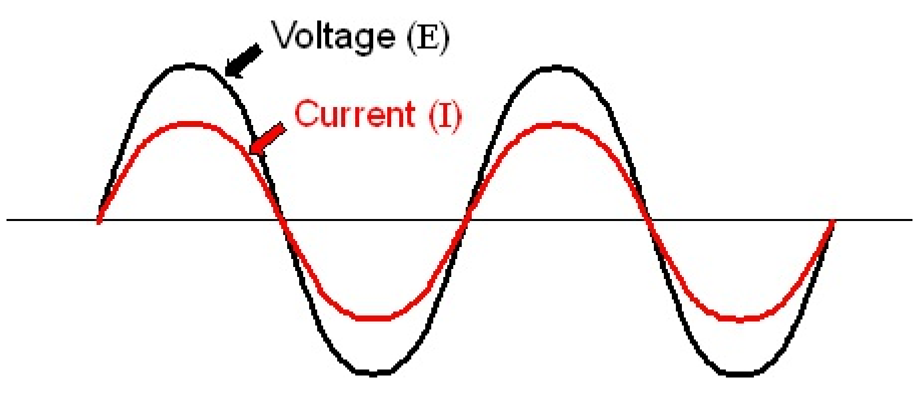


Figure 226.1 *In the case of a resistive ac load, the voltage sine wave and the current sine wave are in alignment. They are zero at the same time and they reach their peak value at the same time.*

Power in an ac Circuit: In a direct current (dc) circuit power is simply the product of the voltage and current (P = E x I). Because the voltage and current are sine waves in ac circuits the power calculation requires an additional factor added to the power equation to account for the alignment of the voltage and current sine waves. This additional factor is the *power factor* (pf). *The power equation for an ac circuit is the voltage times the current times the power factor (P = E x I x pf), Equation 226.1 .*

$$P = E \times I \times pf$$ **Eq. 226.1**

Since the voltage and current in an ac circuit are in the form of a sine wave the level of voltage and current are constantly changing in magnitude. What is measured from an ac circuit as voltage or as current is known as the "***effective***" value of the voltage and current. The effective value is often called the "***rms***" (root mean square) value of the voltage or current. The effective or **rms** value is what is used in all calculations. To understand more about effective value refer to *Tech Note 205.*

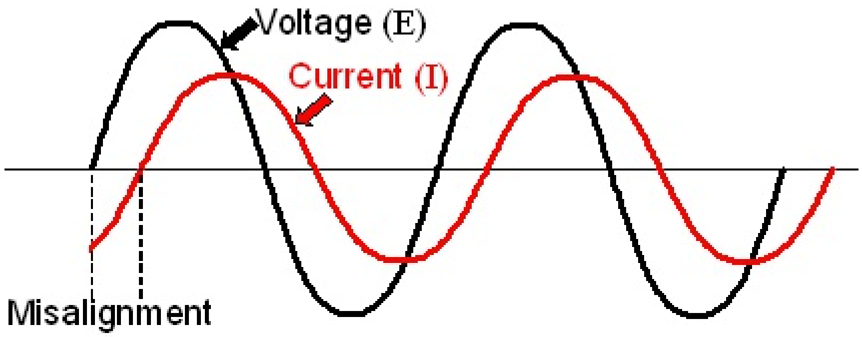


Figure 226.2 *In the case of an ac circuit powering an electric motor the voltage sine wave and the current sine wave are not aligned. Power factor is a value that indicates the amount of misalignment.*

Calculating Power of an ac Circuit: An electric water heater is an example of what is known as a resistive ac load. In this case the voltage sine wave and the current sine wave will be in alignment and the power factor (pf) will have a value of 1.0. As an example, assume an electric water heater operates at 240 volts and has a current draw of 14.6 amperes. Determine the power rating of the water heater.

$$P = 240V \times 14.6A = 3504W$$

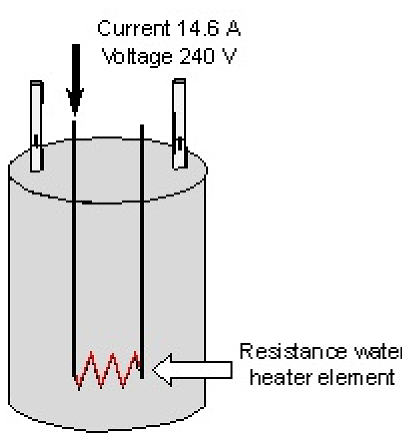


Figure 226.3 *The water heater is a resistance load so the power factor is 1.0. The product of the voltage and current for this water heater is 3504 watts.*

Here is an example of an ac load where the power factor (pf) is not 1.0 and the current and voltage sine waves are out of alignment with each other (*Figure 226.4*). A common example is an electric motor. In this case the motor is single-phase 3-horsepower and operates at 240 volts and draws 17 amperes. Assume the power factor (pf) has a value of 0.75. The task is to determine the wattage draw of this electric motor.

$$P = 240V \times 17A \times 0.75 = 3060W$$

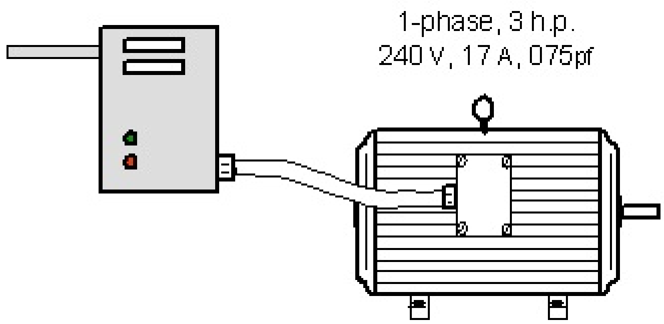


Figure 226.4 *An electric motor supplied with ac power is an example of a load where the power factor (pf) has a value of less tan 1.0. This is a single-phase motor that draws 17 amperes at 240 volts with a power factor of 0.75.*

Power factor (pf) of a load must be given when performing a power calculation. For an actual situation both power factor and power can be easily measured directly. In the case of loads such as electric motors the value of the power factor will change as the load on the motor changes. When power factor is given on a motor nameplate that value was determined under test conditions and should be fairly accurate when the motor is operating a full load.

Effective or rms Value of a Sine Wave: A sine wave such as the one shown in *Figure 226.5* will have two important values. One is the peak value of the sine wave EP. This peak value is both positive and negative in a single cycle. The value of the sine wave that is used for calculations is the effective voltage or sometimes called the “rms” voltage. If the sine wave for a load is not distorted the ratio of the peak voltage divided by the effective voltage is the square root of the number 2 which has a value of 1.414. If a voltmeter measures the voltage at a wall outlet to be 120 volts multiply that value by 1.414 to find the peak value. (120 V x 1.414 = 170 V). The peak voltage, when the effective voltage is 120, will be 170 volts. To find the effective voltage when the peak voltage is given just divide the peak voltage by 1.414.

$$E_{effective} = \frac{E_{peak}}{1.414} \qquad \textbf{Eq. 226.2}$$

$$E_{peak} = E_{effective} \times 1.414 \qquad \textbf{Eq. 226.3}$$

Frequently the waveform of a circuit is not a perfect sine wave. Many ammeters are only capable of reading an accurate value of the current when the ac current is a perfect sine wave. To always get an accurate reading when measuring current, be sure on the face of the ammeter are the words “True rms.” Meters with this designation will give an accurate reading even with the current sine wave is distorted.

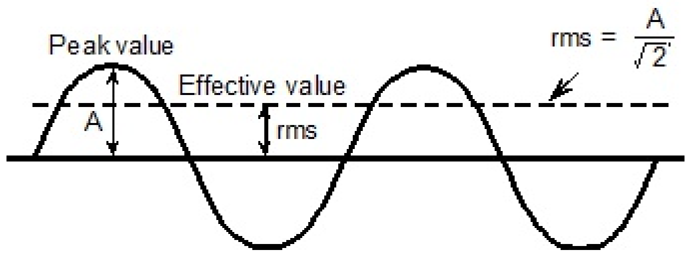


Figure 226.5 *Divide the peak value by 1.414 to get the effective (rms) value of the sine wave.*

Calculating 3-Phase Power: A single-phase load is supplied with only two wires but a 3-phase load is supplied with three wires. The 3-phase device is supplied three separate sine waves each off-set from the others by 120^o of angle which is time of about 5.6 milli-seconds (ms). Each of these sine waves do not reach their peak at the same time and they do not become zero at the same time. Fortunately the equation to calculate 3-phase power only requires one additional factor added to the single-phase power equation. That factor is the square root of the number 3 which has a value of 1.73. The 3-phase power equation is shown in *Equation 226.4.* It is the **voltage** time the **current** times the **power factor** (pf) times **1.73**.

$$P_{3-Phase} = 1.73 \times E \times I \times pf$$ **Eq. 226.4**

Even though a 3-phase load has three wires instead of two, the current draw of an identical load will be less for 3-phase as compared to single-phase. This is one of several advantages to 3-phase as compared to single-phase. Lets examine two important types of loads, both single phase and 3-phase. This first set of examples deal with a resistance type load where the power factor (pf) can be assumed to be close to 1.0. The examples are 5000 watt, 240 volt electric resistance heaters. One is single-phase and the other is 3-phase. The task is to calculate the current draw of each electric heater.

$$I_{1-Phase} = \frac{P}{E \times pf}$$ **Eq. 226.5**

$$I_{3-Phase} = \frac{P}{1.73 \times E \times pf}$$ **Eq. 226.6**

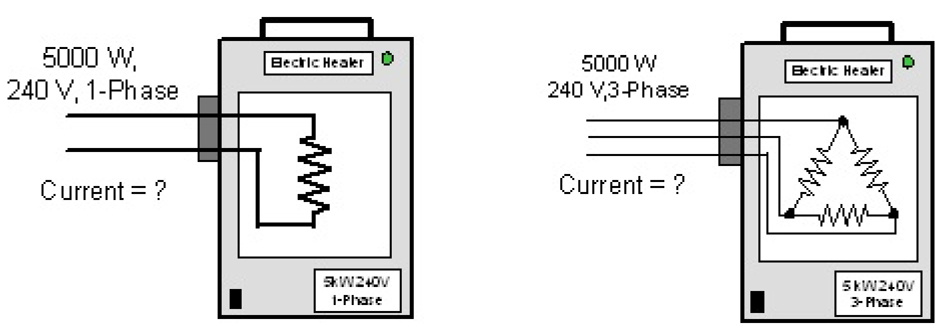


Figure 226.6 *Determine the current draw of a 5 kW, 240 volt single-phase resistance heater and a 5 kW, 240 volt 3-phase resistance heater and compare the two currents. (Calculations on next page)*

$$I_{1-Phase} = \frac{5000W}{240 \times 1.0} = 20.8A \qquad I_{3-Phase} = \frac{5000W}{173 \times 240V \times 1.0} = 12.0A$$

Notice the current for the 3-phase heater is much lower which means the conductor size for the circuit will be smaller. Next is an example where the power factor (pf) is a value less than 1.0 (0.8). Both electric motors are rated 10 horsepower and operate at 240 volts. One is single-phase and the other is 3-phase. Notice the 3-phase motor is smaller and draws less current.

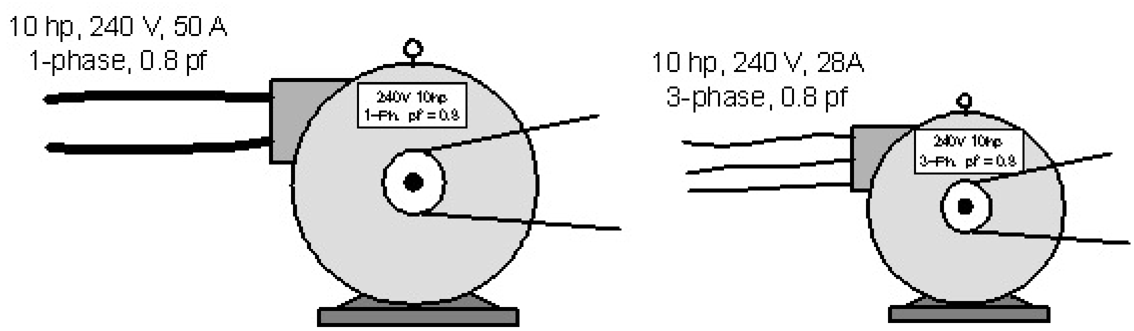
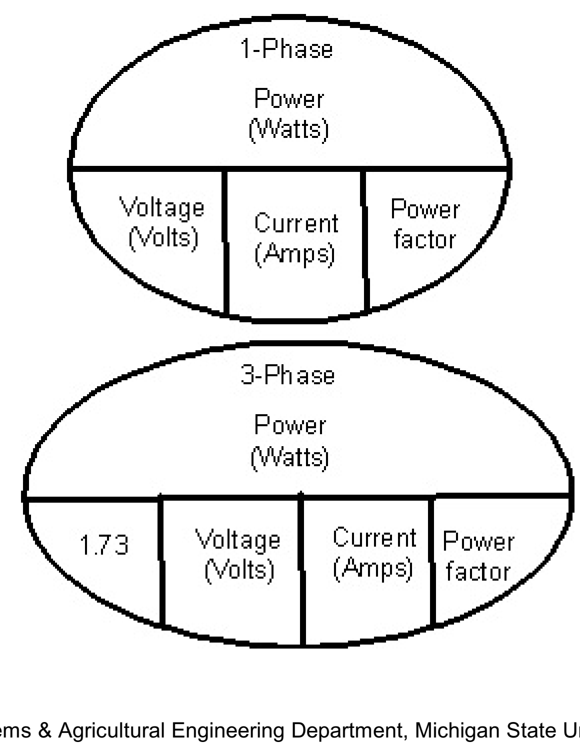


Figure 226.7 *This example is of two 10 horsepower, 240 volt electric motors one that is single-phase and the other 3-phase. Each has a power factor of 0.8 but the single-phase motor draws 50 amperes and the 3-phase motor only draws 28 amperes. The task is to calculate power draw of each motor.*

The diagrams below are an easy way to remember the power equations.

$$P_{1-Phase} = 240V \times 50A \times 0.8 = 9600W$$
$$P_{3-Phase} = 1.73 \times 240V \times 28A \times 0.8 = 9300W$$



Apparent Power and Real Power: There are two terms that describe power. One is apparent power and the other is real power. When the current sine wave and the voltage sine wave are perfectly in alignment such as shown in *Figure 226.1*, the apparent power and the real power have the same value and the power factor has a value of one (1.0). With inductive reactance or capacitive reactance in the circuit such as the case with an electric motor, a portion of the current is needed to create the magnetic field in the motor windings with the remaining portion of the current needed to produce the power. When this is the case the power factor decreases to a value less than one (< 1.0). Apparent power is simply the voltage times the current called volt-amperes or VA. In order to determine the actual power, the product of voltage and measured current must be multiplied by the power factor (pf). The inverse cosine of the power factor is the angle the voltage and current sine waves are out of alignment such as shown in *Figure 226.2.* The power the utility records is the real power.

The real power is the product of the voltage and current times the power factor (pf) and is measured in watts. Apparent power is simply the product of volts and amperes and it's unts is simply volt-amperes or "VA." When drawn as vectors there is an angle between the real power on the horizontal axis and the apparent power which becomes the hypotenuse of a right triangle. The angle between the real power an apparent power is determined by taking the inverse cosine of the power factor (angle = $\cos^{-1}$ pf).

Example: A single-phase electric motor is powered at 240 volts, draws 12 amperes with a power factor (pf) of 0.85. The real power and apparent power are shown in the vector diagram of *Figure 226.8.*

Real Power = 240 V x 12 A x 0.85 = 2448 watts

Apparent power = 240 V x 12 A = 2880 VA

$\theta = \text{Cos}^{-1} 0.85 = 32^{o}$

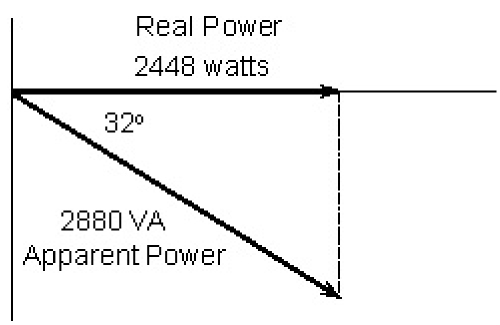


Figure 226.8 *The real power and apparent power form a right triangle with real power as the horizontal leg and the apparent power as the hypotenuse. The angle between the real power and the apparent power is the inverse cosine of the power factor.*