# Electrical Tech Note — 317

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**Michigan State University**

### Parameters Influencing Magnetic Flux Density

Electrical current completes a circuit from one terminal of a source and back to the other terminal of the source. The current will seek a path that has the least resistance. Current will flow in all paths, but there will be more current flowing in paths with little resistance. A magnetic flux will complete a circuit from a bar magnet or an electromagnet from the North pole to the South pole. Some materials such as iron and steel are good conductors for a magnetic flux and other materials such as air are poor conductors for a magnetic flux. **Reluctance** is the quantity that opposes magnetic flux. A material that is a good conductor for a magnetic flux has a low reluctance. The units of reluctance is amperes per weber which can be considered the current required to obtain a weber of magnetic flux. The opposite of reluctance is **permeability**. A material with a high permeability is a good path for magnetic flux.

**Magnetic Circuit:** When creating a strong magnetic flux density (**B**) with an electromagnet, the goal is to achieve as high a flux density as possible with the lowest magnetic field intensity. Magnetic field intensity (**H**) is the number of amperes per unit length of a magnetic circuit such as amperes per inch or amperes per meter. A core through the center of the electromagnet and completing a loop around the outside is needed with as high a permeability (**μ**) as possible in order to maintain a high magnetic flux density. If a path of high permeability is provided, the magnetic flux will be concentrated in the magnetic material and not spread out into space. The permeability of some common magnetic path materials is shown in Figure 317.1. The materials with high permeability have a drawback called **saturation**. A point is reached where the magnetic flux density does not increase when magnetic field intensity is increased.

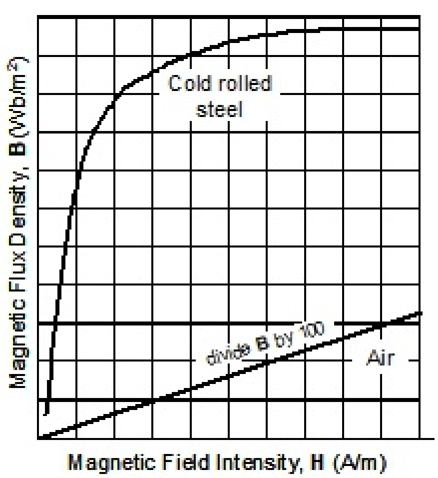


**Figure 317.1** Magnetic flux density (**B**) increases in a material as the magnetic field intensity (**H**) is increased, but the materials with the best magnetic properties reach a point called **saturation** where an increase in magnetic field intensity results in only a minimal increase in magnetic flux density.

Air and a vacuum as a path for a magnetic flux do not saturate, but they have very low permeability. The magnetic flux spreads out and is not concentrated. A high flux density is hard to achieve in air. Similar to insulation around an electrical conductor that prevents loss of current from the conductor, most of the magnetic flux will remain in a steel path with little flux **leakage** into the surrounding air. The flux is then concentrated in the steel and a high flux density can be achieved. The ratio of permeability of magnetic path materials to air is only a few thousand to one, so there will be some flux leakage to the surrounding air.

Although copper and aluminum are excellent electrical conductors, they make very poor conductors for a magnetic flux. Materials made with iron make the best conducting medium for a magnetic flux, such as cold rolled steel.

**Magnetic Permeability:** The symbol for magnetic permeability is the Greek letter mu (μ) and the units are Henry per meter (H/m). The Henry is the unit of measure of inductance and is equal to one volt-second per ampere (V·s/A). The **permeability of free space** or a vacuum ($\mu_s$) is equal to $1.256 \times 10^{-6}$ H/m. It is customary to describe permeability of materials relative to the permeability of free space. Approximate values of **relative permeability** ($\mu_r$) are 1.0 for air, 1000 for cast steel, 4000 for sheet steel, and 5000 for cast iron. The permeability of a magnetic path is determined by multiplying free space permeability by the relative permeability ($\mu_r \times \mu_s$).

**Magnetic Flux:** It was discovered that when fine particles of iron were placed near a magnet, the particles would stick together and line up between the ends of the magnet in strings or lines. This is illustrated in Figure 317.2. Even though it was later discovered that actual lines do not exist, it was convenient to refer to a magnetic flux as "lines of force". Magnetic flux (φ) can be quantified, and today the common unit of magnetic flux is the Weber (Wb). Another unit of magnetic flux is the Maxwell and it takes $10^8$ Maxwell to equal one Weber. Compared to the concept of lines of force, 1 Weber is equal to 100 million lines ($10^8$). Magnetic flux has direction and it completes a magnetic circuit. The direction of a magnetic flux outside of the magnet is considered as pointing from the North pole to the South pole of the magnet as shown in Figure 317.2. A Weber is also equal to one volt-second (Wb = V·s). Webers and Henrys will sometimes both be found in the same formula. One Henry is one Weber per ampere (H = Wb/A), or one Weber is equal to one Henry-ampere (Wb = H·A). Be careful with the use of the letter "H" in the following analysis. The unit Henry and variable magnetic field intensity are both represented by the letter "H".

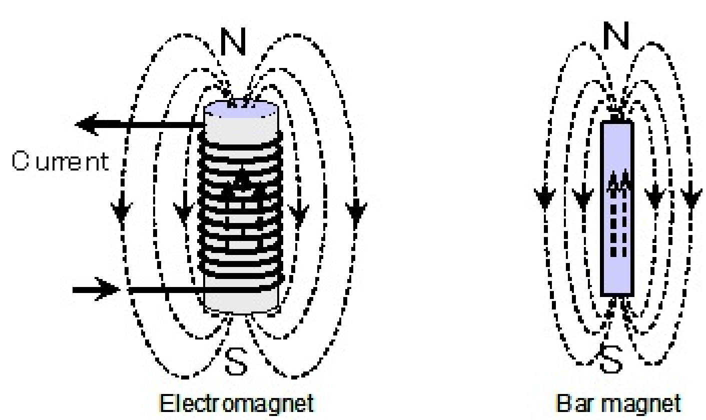


**Figure 317.2** Whether a permanent magnet or an electromagnet, a flux resembling lines develop, completing a circuit from North to South outside of the magnet and from South to North inside the magnet.

Magnetic flux does not require some physical material to complete a circuit. It can act through free space (vacuum). The behavior of a magnetic flux in air and many nonmetallic materials acts essentially the same as a magnetic flux in free space. Air is a poor medium for a magnetic flux. The flux spreads out when it must act through air or free space, and even though a strong magnet may be used, at any point away from the magnet the strength of the flux may be weak. It was discovered that iron and steel provide an excellent path for a magnetic flux.

**Terminology and Units:** An analogy can be drawn between current flow in a circuit and flux in a magnetic circuit. Ohm's law states that current flow in a circuit is equal to the voltage divided by the resistance. For a magnetic circuit, magnetic flux ($\varphi$) is analogous to current, a quantity called magnetomotive force or mmf (F) is analogous to voltage, and reluctance of the magnetic circuit (R) is analogous to resistance of an electrical circuit. The magnetic circuit formula analogous to Ohm's law is that the amount of flux ($\varphi$) is equal to the magnetomotive force (F) divided by the reluctance (R) of the magnetic circuit. In the real world the calculations can get quite complex. The following discussion is intended to gain a basic understanding of magnetic principles, and assumptions have been made to simplify the concepts and calculations. In many cases, however, these simplifications are valid and can be made to obtain approximate solutions to real problems.

Electrical current flow (I) is measured in amperes (1 Ampere = 1 Coulomb/second) and a magnetic flux is produced around a conductor through which current is flowing. The magnetic flux intensity is proportional to the current flow.

I = current flow in the coil (A)

The unit of measure of length for a magnetic flux circuit is in meters. The cross-sectional area of the magnetic flux path perpendicular to the direction of the flux is in square meters. For the purpose of the following examples, the mean length of the magnetic flux circuit will be used, and it will be assumed the flux is evenly distributed across the cross-section of the flux path.

L = length of the magnetic circuit (m)

A = cross-sectional area of the magnetic circuit ($m^2$)

A strong magnetic flux can be created by passing current through wire tightly wound into a coil as shown in Figure 317.2. The magnetic flux of each wire will add to form a strong magnetic flux. An important parameter for an electromagnet is the number of turns of wire (N) forming the coil.

N = number of turns of the coil (no units)

The excitation required to produce a magnetic flux by passing current through a coil is called the magnetomotive force (F) which is in units of total amperes flowing in a circle around a coil. The units ampere-turns is often used, but turns is not really a unit. Turns are necessary to increase the amperes flowing in a circular path. Magnetomotive force (mmf) is the total current of the coil which is an accumulation of the current in each turn or winding as indicated by Equation 317.1. Tightly winding this coil to a core material will result in a high level of linkage of this magnetomotive force to the core material. This magnetomotive force is exerted over the total length of the magnetic circuit.

$F = N \times I$ <u>magnetomotive force</u> (A or Ampere·Turns) **Equation 317.1**

A magnetic circuit extends from one pole of a magnet, through some medium and back to the other pole, and then through the magnet to the original pole. Magnetic flux, like electrical current, completes a circuit back to the source. When air or free space is the medium surrounding a magnet, the length and cross-sectional area of the flux path is limitless. Experimental results can be used to describe the distribution of the flux in air about a magnet. When a material such a iron or steel is used as the path for the magnetic flux, most of the flux will be confined within the material and the magnetomotive force is considered to be acting over the total length of the magnetic circuit. The intensity of a magnetic field will be greater when the magnetic circuit is shorter. Every material offers at least some resistance (called reluctance for magnetic materials) to a magnetic flux, and the shorter the path the higher the resulting intensity. It is important to know the magnetic field intensity (H) which is the magnetomotive force (F) divided by the length (L) of the magnetic circuit. Since magnetomotive force is in amperes, and magnetic circuit length is in meters, magnetic field intensity is in amperes per meter as indicated by Equation 317.2.

$$H = \frac{F}{L}$$ magnetic field intensity (A/m) **Equation 317.2**

Magnetic permeability is a measure of how good a material is as a path for a magnetic flux. The higher the permeability the better the material is as a path for a magnetic flux. Generally permeability is expressed as a number relative to the magnetic permeability of free space. To determine the actual permeability of a material ($\mu$), the relative permeability of the material ($\mu_r$) is multiplied by the magnetic permeability of free space ($\mu_s$) as described by Equation 317.3. The unit of permeability is the Henry per meter (H/m). One Henry of inductance is equal to one volt being produced by a time rate of change of current of one ampere per second. The units of a Henry are volt seconds per ampere.

$$\mu = \mu_r \times \mu_s$$ magnetic permeability (H/m) also (Wb/A·m) **Equation 317.3**

$\mu_r$ relative permeability (no units)
$\mu_s$ permeability of free space ($1.256 \times 10^{-6}$ H/m)

Frequently an objective is to determine the magnetic flux density at a point in a magnetic circuit, or to determine the current level and number of turns needed at a coil to produce a particular magnetic flux density at a point in a magnetic circuit. The larger the permeability of the magnetic circuit, the smaller the current required to produce the flux density required. The unit of magnetic flux density is the Weber per square meter (Wb/m$^2$). Magnetic flux density can be determined by multiplying the core permeability ($\mu$) by the magnetic field intensity (H) as shown in Equation 317.4.

$$B = \mu \times H$$ Magnetic flux density (Wb/m$^2$) **Equation 317.4**

The total magnetic flux is equal to the magnetic flux density (B) times the cross-sectional area of the magnetic circuit which is usually the cross-sectional area of the magnetic core, shown by Equation 317.5. The second of the following formulas is similar to Ohm’s law for electrical circuits. The total magnetic flux is equal to the magnetomotive force divided by the total reluctance of the magnetic circuit as shown in Equation 317.6. This is a useful formula for analyzing magnetic circuits that consist of a magnetic core and one or more air gaps. For some magnetic circuits, there may be a portion of the circuit that forms a parallel path such as shown in Figure 317.3. The sum of the magnetic flux in each parallel path will add up to the total magnetic flux. It is common to have two identical parallel paths completing a magnetic circuit, in which the magnetic flux will divide equally with half of the flux in each parallel path.

$\varphi = B \times A$ Total magnetic flux (Wb) **Equation 317.5**

$$\varphi = \frac{F}{R} \quad \text{(Wb)}$$ **Equation 317.6**

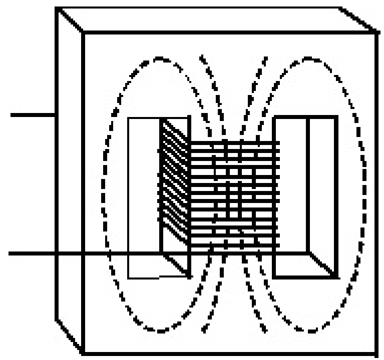


**Figure 317.3** Some magnetic cores are constructed with a parallel path and the flux divides to take both paths available.

Reluctance is the resistance to magnetic flux within a magnetic circuit. Like resistance in an electrical circuit, the total reluctance of a magnetic circuit with different dimensions or with different core materials in series is equal to the sum of the reluctance of each section of the magnetic circuit. Assuming the magnetic flux is uniformly distributed across the magnetic circuit material, the reluctance of the circuit is proportional to the length of the circuit and inversely proportional to the cross-sectional area of the material. The unit of reluctance is the reciprocal of Henrys. Reluctance is also amperes per Weber. Be careful with the letter “A” which represents area in a formula and amperes in the dimensions. The total reluctance of a core represents the amperes required to achieve one weber of flux. Equation 317.7 is used to determine the reluctance of each section in series that makes up a magnetic core. The total reluctance is the sum of each section’s reluctance. The following example shows how the magnetic flux density of a core with uniform dimensions is determined.

$$R = \frac{L}{\mu \times A} \quad \text{reluctance} \quad \text{(1/H) also (A/Wb)}$$ **Equation 317.7**

**Example 1:** The following example will illustrate how magnetic flux density (B) can be determined for an electromagnet coil around a steel core that makes a complete magnetic circuit as illustrated in Figure 317.4. The mean length of each side of the steel core is 12 cm. The cross-section of the steel core is 4 cm on each side. There are 400 turns of wire around one leg of the core with a current of 0.2 amperes. The relative permeability of the core material ($\mu_r$) is 4000. The following assumptions can be made in order to obtain an approximate result without difficult calculations. It will be assumed that all of the magnetic flux produced by the coil will be placed into the core (flux and coil are linked). There will be no core leakage (all flux remains within the core), and the density of the flux across the core cross-section is uniform.

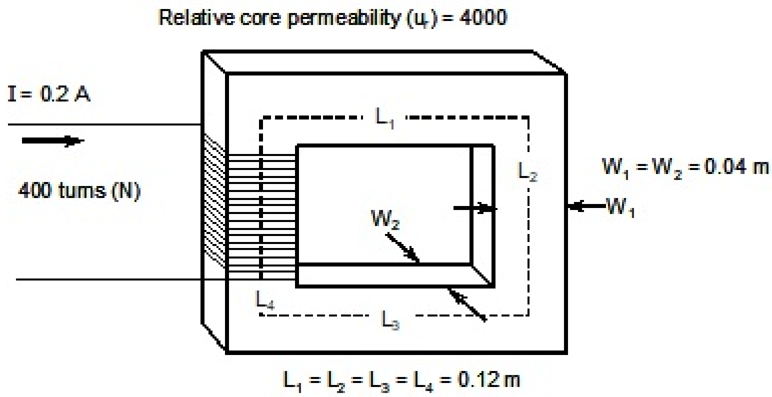


**Figure 317.4** Determine the magnetic flux density (B) for a steel core with relative permeability ($\mu_r$) of 4000 and 0.2 amperes flowing in a 400 turn coil.

**Solution:** First determine the magnetomotive force (F)

$$F = N \times I = 400 \text{ turns} \times 0.2 \text{ A} = 80 \text{ A-turns}$$

Next determine the magnetic field intensity (H). The mean magnetic circuit length is 0.48 m.

$$L = L_1 + L_2 + L_3 + L_4 = 4 \times 0.12 \text{ m} = 0.48 \text{ m}$$

$$H = \frac{F}{L} = \frac{80 \text{ A}}{0.48 \text{ m}} = 167 \text{ A/m}$$

Finally determine the magnetic flux density (B), but first determine the permeability of the magnetic circuit.

$$\mu = \mu_r \times \mu_s = 4000 \times 1.256 \times 10^{-6} \text{ H/m} = 5.029 \times 10^{-3} \text{ H/m (Wb/A·m)}$$

$$B = \mu \times H = 5.029 \times 10^{-3} \text{ Wb/A·m} \times 167 \text{ A/m} = 0.839 \text{ Wb/m}^2$$

**Air Gap in a Magnetic Circuit:** Many magnetic circuits have an air gap. An example is an electric motor where a magnetic circuit must be completed through a rotor. A solenoid is another example of a magnetic circuit with an air gap. The permeability of air is very low compared to iron or steel and the air gap represents a significant reluctance in the magnetic circuit even though the length of the air gap is very small. Since air surrounds the magnetic core as well as in the gap, the magnetic flux tends to expand outward at the gap as shown in Figure 317.5. This expanding of the magnetic flux at the air gap is called **fringing**. The effective cross-sectional area of the air gap as a magnetic circuit is larger than the cross-sectional area of the magnetic core as indicated by Equation 317.8. The flux density across the air gap is not uniform, but some assumptions can be made for the purpose of analysis that have been found to give a reasonable result. The flux density is assumed to be uniformly distributed across a cross-section that is equal to the products of the pole dimensions ($w_1$& $w_2$) increased by the length of the air gap ($L_g$).

$$\text{Air gap cross-section} = (w_1 + L_g) \times (w_2 + L_g) \qquad \textbf{Equation 317.8}$$

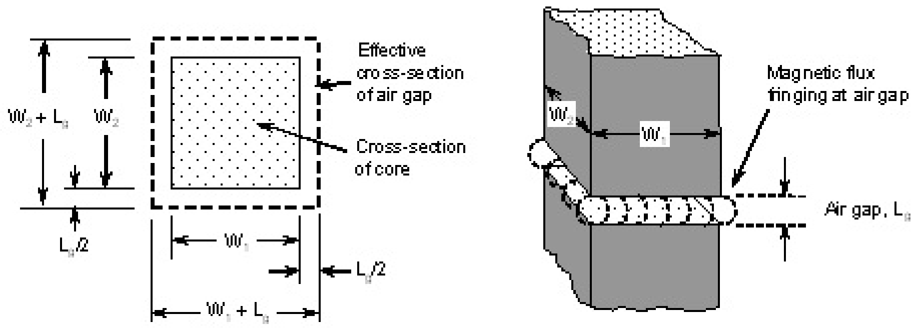


**Figure 317.5** Fringing of the magnetic flux at an air gap will occur and for analysis purposes, the cross-sectional area of the flux through the air gap can be assumed to be the product of the dimensions of the core increased by the length of the air gap ($L_g$).

**Example 2:** The following example will illustrate how magnetic flux density (B) can be determined for an electromagnet coil around a steel core that makes a magnetic circuit with an air gap of 0.002m as illustrated in Figure 317.6. The mean length of each side of the rectangular magnetic circuit is 12.0 cm. The steel core on the side of the magnetic circuit with the air gap is reduced to a length of 0.059 m. The cross-section of the steel core is 4.0 cm on each side. There are 400 turns of wire around one leg of the core with a current of 0.2 amperes. The relative permeability of the core material ($\mu_r$) is 4000. The following assumptions can be made in order to obtain an approximate result without difficult calculations. It will be assumed that all of the magnetic flux produced by the coil will be placed into the core (flux and coil are linked). There will be no core leakage (all flux remains within the core except at the air gap), and the density of the flux across the core and air gap cross-section is uniform.

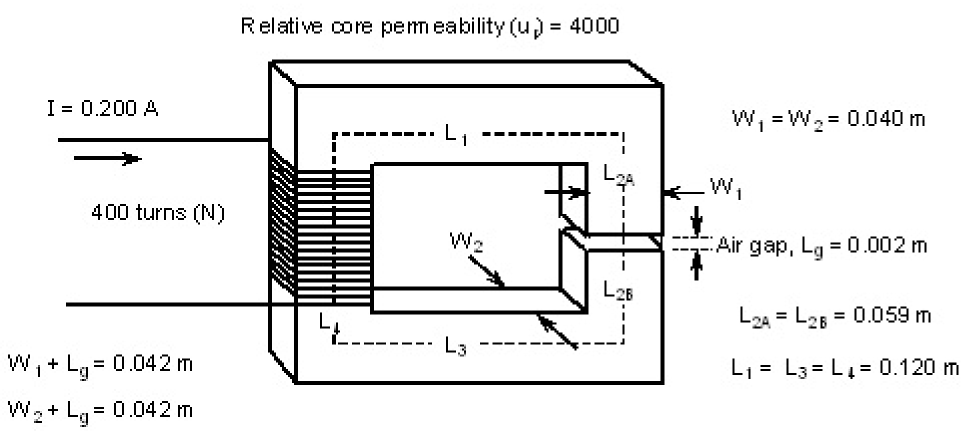


**Figure 317.6** A magnetic circuit consists of a rectangular steel core with 400 turns on one leg and a current flow of 0.2 amperes. One leg of the steel core has an air gap with a length of 0.002m.

**Solution:** When determining the flux density (B) with a magnetic circuit consisting of sections in series that do not have the same permeability, the use of reluctance rather than permeability is a more efficient method of analysis. The reluctance can be determined for the steel core and for the air gap with the total magnetic circuit reluctance being the sum of the reluctance of each section in series.

**Solution:** First determine the magnetomotive force (F)

$$F = N \times I = 400 \text{ turns} \times 0.2 \text{ A} = 80 \text{ A-turns}$$

Next determine the dimensions of the sections of the magnetic circuit, the magnetic permeability of each section, and then the reluctance of each section of the magnetic circuit.

Steel core

$$L_C = L_1 + L_{2A} + L_{2B} + L_3 + L_4$$
$$= 0.120\text{m} + 0.059\text{m} + 0.059\text{m} + 0.120\text{m} + 0.120\text{m} = 0.479\text{m}$$

$$A_C = w_1 \times w_2 = 0.040\text{m} \times 0.040\text{m} = 1.60 \times 10^{-3} \text{ m}^2$$

$$\mu_C = \mu_r \times \mu_S = 4000 \times 1.256 \times 10^{-6} \text{ H/m} = 5.029 \times 10^{-3} \text{ H/m}$$

$$R_C = \frac{L_C}{\mu_C \times A_C} = \frac{0.479\text{m}}{5.029 \times 10^{-3} \text{ H/m} \times 1.60 \times 10^{-3} \text{ m}^2} = 0.059 \times 10^6 \text{ 1/H}$$

Air gap

$$L_g = 0.002\text{m}$$

$$A_g = (w_1 + L_g) \times (w_2 + L_g) = (0.040\text{m} + 0.002\text{m}) \times (0.040\text{m} + 0.002\text{m})$$
$$= 1.76 \times 10^{-3} \text{ m}^2$$

$$\mu_g = \mu_r \times \mu_S = 1 \times 1.256 \times 10^{-6} \text{ H/m} = 1.256 \times 10^{-6} \text{ H/m}$$

$$R_g = \frac{L_g}{\mu_g \times A_g} = \frac{2 \times 10^{-3} \text{ m}}{1.256 \times 10^{-6} \text{ H/m} \times 1.76 \times 10^{-3} \text{ m}^2} = 0.905 \times 10^6 \text{ 1/H}$$

Total reluctance $R_T$

$$= R_C + R_g = 0.059 \times 10^6 \text{ 1/H} + 0.905 \times 10^6 \text{ 1/H}$$
$$= 0.964 \times 10^6 \text{ 1/H also (A/Wb)}$$

Next determine the total flux (φ) for the magnetic circuit.

$$\varphi = \frac{F}{R_T} = \frac{80 \text{ A}}{0.964 \times 10^6 \text{ A/Wb}} = 82.97 \times 10^{-6} \text{ Wb}$$

Finally determine the magnetic flux density (B) of the magnetic circuit with the air gap. The following flux density is determined for the steel core. If a value was determined for the air gap it would only be an approximate average value. The flux density across the cross-section of the air gap is not uniform. Compared with the previous example where there was no air gap, the flux density of the same core was 0.839 Wb/m$^2$. Even a small air gap causes a major reduction in the flux density of a magnetic circuit.

$$B = \frac{\varphi}{A_C} = \frac{82.97 \times 10^{-6} \text{ Wb}}{1.60 \times 10^{-3} \text{ m}^2} = 0.052 \text{ Wb/m}^2$$

**Magnetic Core Construction:** When an electromagnetic coil is energized with alternating current, the magnetic flux is constantly in motion and cutting across the magnetic core. Since the magnetic core is usually steel or iron and an electrical conductor, a current will be induced to flow in the core. This core current flow results in loss of energy and undesirable heat produced. To minimize current flow in the magnetic core, the core of many devices such as solenoids, transformers, and motors consist of thin sheets of steel with a very thin electrical insulating coating similar to the diagram in Figure 317.7. Unfortunately the insulation has a low permeability and creates a thin gap between the sheets of steel which causes a slight decrease in the overall permeability of the core, but it is necessary to prevent excessive heat and losses.

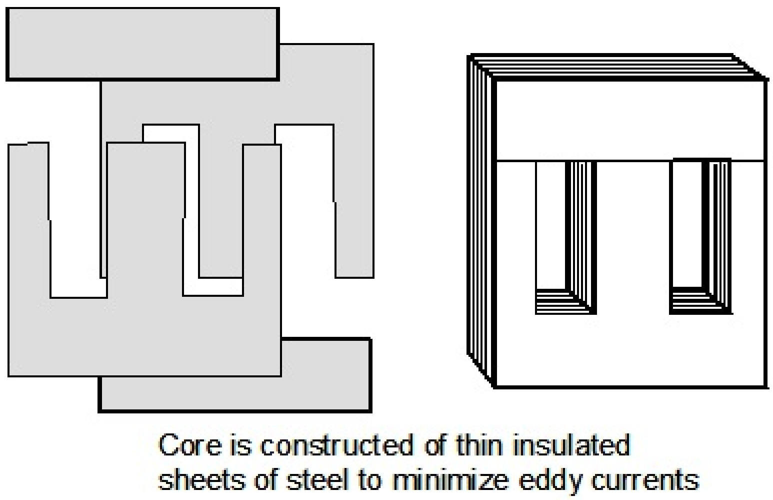


**Figure 317.7** A magnetic core is generally constructed of thin insulated sheets of steel to minimize eddy currents in the core.

**Hysteresis:** A physical alignment occurs within a material that becomes magnetized. An effort or energy is required to magnetize a material. The magnetic polarity of a material such as steel can be changed and the material magnetized with the opposite polarity. This process requires a physical reversal within the material. The material will resist magnetic reversal. A magnetizing force can be applied to a magnetic material such as steel and the steel will become magnetized. If the magnetizing force is removed, the steel will remain magnetized to some extent. A physical alignment has occurred in the steel and it continues to act as a magnet. In order to completely de-magnetize the steel, a magnetizing force of opposite polarity must be applied. This property to resist change is called **hysteresis**. If the magnetic flux of the steel is plotted through a complete cycle of magnetizing first with one polarity, and then of the opposite polarity, and then back to the original polarity, the magnetic flux density verses magnetic field intensity (H) is shown in Figure 317.8. A loop is created with some finite area in the center. This is known as a hysteresis loop. Note in Figure 317.8 there is no point where magnetic field intensity (H) and magnetic flux density (B) are both zero. If there was no hysteresis, the graph would be a single line through the origin of the graph.

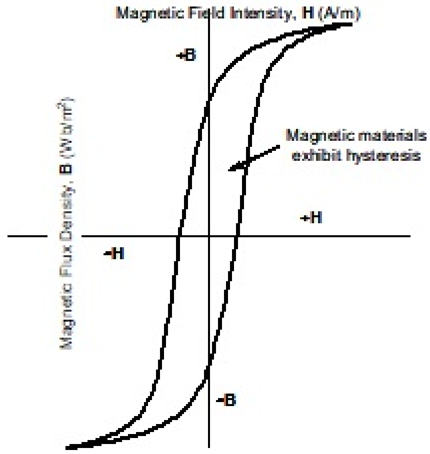


**Figure 317.6** A material such as steel does not become completely de-magnetized when the magnetic field intensity goes to zero. It will take some reverse magnetic field intensity to completely de-magnetize the steel. This graph of a complete magnetizing cycle of a material like steel is called a hysteresis loop.

**Magnetic Circuit Analysis:** Care must be exercised when performing calculations with magnetic circuits in that some of the variables have the same letter symbols and units. Table 317.1 gives common terms, symbols and units used in magnetic circuit calculations. A magnetic field forms a connection between electrical energy and mechanical energy. Magnetic fields are generated by electrical charge in motion. Likewise, a magnetic flux moving across an electrical conductor will cause charge to move in the conductor. A force (f) exerted on a charge (q) moving at a velocity (u) in a magnetic flux with density (B) is given Equation 317.9.

$$\mathbf{f} = (q \cdot \mathbf{u}) \times \mathbf{B} = q \cdot u \cdot B \cdot \sin\theta \text{ with} \qquad \textbf{Equation 317.9}$$

θ is the angle between u and B.

where force (f), velocity (u) and flux density (B) are all vectors and the X represents the cross product. The force is acting perpendicular to the plane created by the velocity (u) and the flux (B).

**Table 317.1** Common units used in magnetic circuits and equivalent units.

| Quantity | Symbol | Name | Units | Units | Units | Units |
|---|---|---|---|---|---|---|
| Inductance | L | Henry, H | H | Wb/A | V•s/A | J•s$^2$/C$^2$ |
| Magnetic flux | φ | Weber,Wb | Wb | V•s | H•A | J•s/C |
| Magnetic field intensity | H | | A/m | | C/m•s | |
| Magnetic flux density | B | Tesla, T | Wb/m$^2$ | | J•s/C•m$^2$ | |
| Magnetic permeability | μ | | H/m | Wb/A•m | V•s/A•m | J•s$^2$/C$^2$•m |
| Magnetomotive force | F | | A | | C/s | |
| (F=φ•R) | | | | | | |
| Reluctance (R=L/μ•A) | R | | 1/H | A/Wb | C$^2$/J•s$^2$ | |
| Force | f | Newton, Nt | | | J/m | |
| Mass | m | kilogram, kg | | | J•s$^2$/m$^2$ | |
| Voltage | E | Volt, V | | | J/C | |
| Current | I | Ampere, A | A | | C/s | |
| | | | | | | |
| Charge | q | Coulomb, C | C | | | |
| Energy | | Joule, J | J | | | |
| Time | t | Seconds, s | s | | | |
| Area | A | | m$^2$ | | | |
| Length | L | | m | | | |
| Magnetic flux (Wb) | | | | | $10^8$ lines | |