Solving Circuit Problems

Typical electrical circuits consist of elements that are connected in series with each other, parallel with each other, and a combination of series and parallel. To solve these problems it is important to know how to use Ohm's law, and the power formula as well as the basic principles of a series and parallel circuit. The procedures discussed in this Tech Note are limited to circuits with only one voltage supply. For the purpose of this discussion, voltage will be represented by the symbol **E**, current by the symbol **I**, and resistance by the symbol **R**. Methods of analyzing more complex circuits are discussed in Tech Note 230.

Series Circuit Rules: A series circuit is one where there is only one path through all of the elements. An example of a series circuit with three components is shown in Figure 215.1. Here are the rules for a series circuit.

- The total current (I_T) for a series circuit is the same at all points in the circuit.

  $$I_T = I_1 = I_2 = I_3$$

- The total voltage (E_T) of a series circuit is the sum of the voltages across each component connected in series.

  $$E_T = E_1 + E_2 + E_3$$

- The total resistance (R_T) of the series circuit is the sum of the resistance of each element connected in series.

  $$R_T = R_1 + R_2 + R_3$$

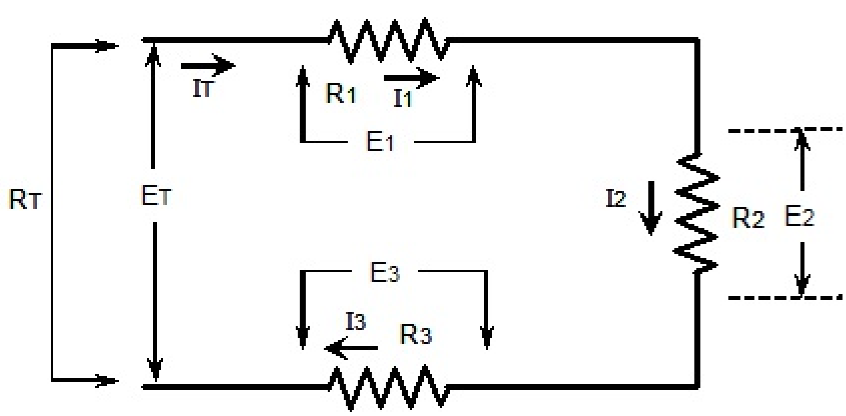

Figure 215.1 *The components are arranges so there is only one path through the circuit with the current the same at every point in the circuit.*

Parallel Circuit Rules: A parallel circuit is one where each component receives the same voltage and the total circuit current is the sum of the current through each component. The current will divide up and take each path available. The paths with the lowest resistance will have the highest current. An example of a parallel circuit is shown in Figure 215.2. Here are the rules for a parallel circuit.

- The voltage is the same across each component of a parallel circuit.

  $$E_T = E_1 = E_2 = E_3$$

- The total current flow (I_T) to components connected in parallel is the sum of the current through each component.

  $$I_T = I_1 + I_2 + I_3$$

- The total resistance (R_T) of a group of components connected in parallel is always less than the value of the component with the smallest resistance.

- The total resistance (R_T) of a group of components connected in parallel can be calculated in several ways depending upon how many resistors are connected in parallel. The generic calculation for total resistance of parallel components is that the reciprocal of the total resistance is equal to the sum of the reciprocal of each parallel resistor as follows:

  $$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

  Determine the lowest common denominator for the fractions and determine the sum. Then invert the value to get the total resistance. If there are only two resistors in parallel, it sometimes is easier to determine the total resistance by a method called product over sum using Equation 215.1.

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2}$$

Equation 215.1

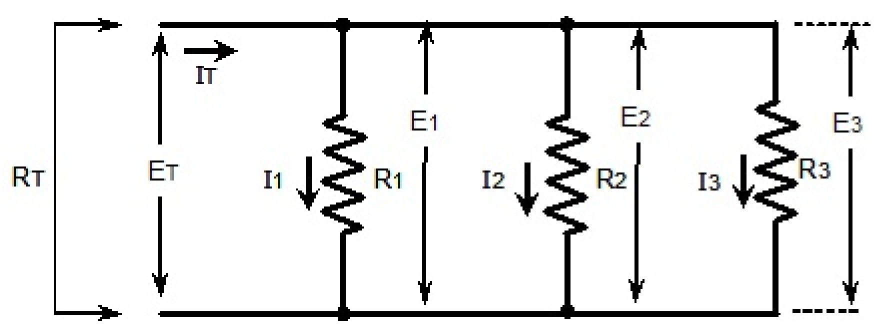

Figure 215.2 *The components are arranges so there are as many paths through the circuit as there are components in the circuit. The voltage is the same across each component and the total current is the sum of the current through each component.*

Solving Circuit Problems: It is important to be able to visualize the circuit and understand which components are connected in series and which are connected in parallel. One technique for visualizing a circuit is to arrange the circuit in a vertical pattern. Put the circuit voltage at the top of the diagram and draw the circuit vertically as shown in Figure 215.3. This is the same as the previous series circuit. Note there is only one path from top to bottom so the current will be the same at any point in the circuit. Now it is logical to see that the total voltage of the circuit will be the sum of the individual series voltages across the components. This technique will help to make complex circuits easier to understand.

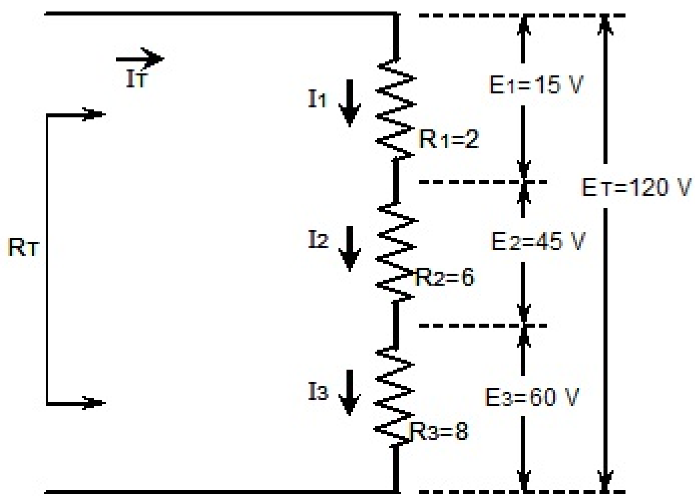

Figure 215.3 *This is a vertical diagram of the circuit of Figure 215.1 where one side of the voltage source is at the top and the other side is at the bottom of the diagram.*

Component Value Box: Sometimes it is easier to determine unknown component values of voltage, resistance, current, and power by placing the known values in a table. Using the component value box, it is easier to see when there are enough values to apply either Ohm's law or the power formula. Table 215.1 is a component value box for the circuit of Figure 215.4. On the right hand side of Figure 215.4 the circuit is redrawn vertically so it is easier to visualize.

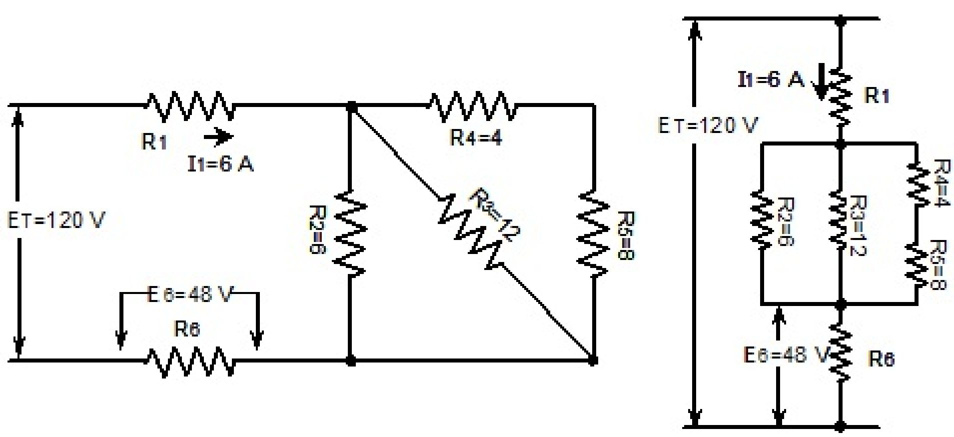

Figure 215.4 *A circuit consists of several components some of which are arranged in series and some are arranged in parallel. The diagram on the right is the same circuit redrawn vertically.*

Table 215.1 This component value box can be used to record the known values to make it easier to determine when Ohm's law or the power formula can be applied.

	Total	1	2	3	4	5	6
E	**120V**	*54V*(6)	*18V*(4, 5)	*18V*(4, 5)	*6V*(9)	*12 V*(9)	**48V**
I	*6A*(1)	**6A**	3A(10)	*1.5A*(10)	*1.5A*(8)	*1.5A*(8)	*6A*(1)
R	*20Ω*(2)	*9Ω*(7)	**6Ω**	**12Ω**	**4Ω**	**8Ω**	*8Ω*(2)
P							

1. The current of 6A through R_1 is the total current and will also pass through R_6.

2. Now the total circuit resistance and the resistance of R_6 can be determined using Ohm's law.

   $R_T = 20\Omega \qquad R_6 = 8\Omega$

3. R_4 and R_5 can be added to get a total resistance of that branch of 12Ω.

4. Note in the center of the circuit are three resistors in parallel. One has a value of 6Ω, another has a value of 12Ω, and the final branch has a value of 12Ω. Apply any parallel resistor technique desired to find that these resistors have a combined value of 3Ω. The circuit can be modified to look like Figure 215.5.

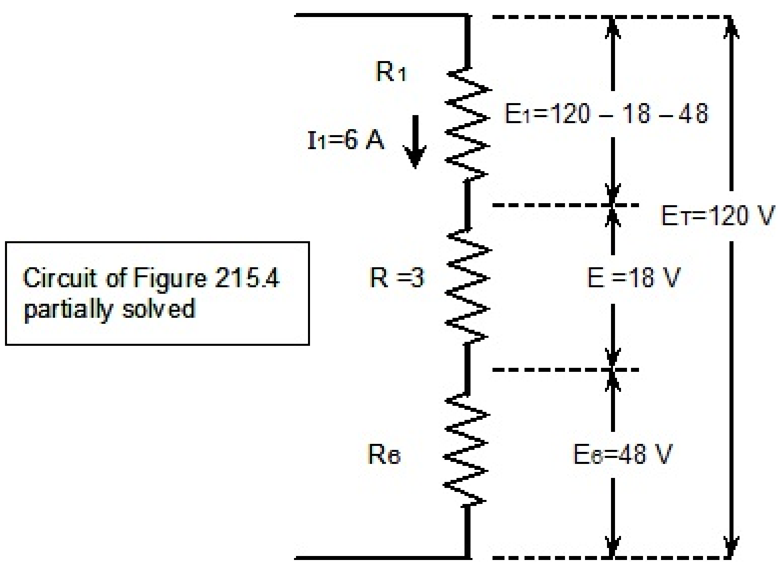

Figure 215.5 *Use the parallel resistor rule to find that a 3Ω resistor in this circuit is equivalent to the original circuit.*

5. It is known that 6A will flow through the parallel section of the circuit, so multiply 6A times 3Ω (Figure 215.5) to determine that the voltage across the parallel section is 18V. The voltage across the 6Ω and 12Ω resistors will be 18V.

6. To find the voltage across R_1 subtract 48V and 18V from the total 120V to get 54 V.

7. Now that the voltage and current are known for R_1, it is possible to calculate the value of R_1 by dividing 54V by 6A to get 9Ω.

8. Now refer back to the right hand diagram of the circuit in Figure 215.4. Resistor R_4 and R_5 are in series and can be combined to be 12Ω. From steps 4 and 5 above it was determined that there is 18V across this 12Ω. Using Ohm's law, find there is 1.5A flowing through this 12Ω resistance.

9. Multiply the current and resistance for each resistor R_4 and R_5 to get the voltage across each resistor, E_4 = 6V and E_5 = 12V. These voltages add up to 18V which is the total voltage across this branch.

10. Finally for R_2 and R_3 divide the voltage by the resistance to get the current, I_2 = 3A, I_3 = 1.5A. Add the current through each of the three parallel branches of Figure 215.4 to get the total current of 6A.

For this circuit power was not a concern so that row in the component value table was not used.

Delta - T Transformation: Sometimes the components of an electrical circuit can be arranged such that the series and parallel reduction techniques do not work. An example is the circuit on the left of Figure 215.6. For the purposes of circuit analysis, the circuit can be changed to an alternate form that functions the same allowing the ability to apply the series and parallel reduction techniques to determine desired unknown values for components of the circuit. Note that the internal components of the circuit on the left of Figure 215.6 form a letter T (R_1, R_2, and R_3). There is a transformation that can convert the three resistors forming the T into three resistors forming a triangle or delta as shown on the right of Figure 215.6 (R_A, R_B, and R_C). The circuit on the right now can be solved using series and parallel reduction techniques. This transformation is known as the T - delta transformation and is accomplished using Equations 215.2.

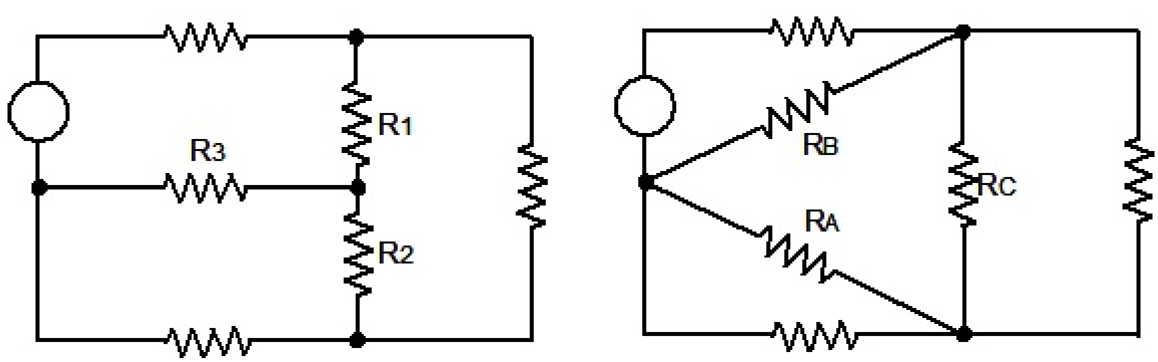

Figure 215.6 *The circuit on the right cannot be solved using typical series/parallel techniques, therefore, the components forming the T have been transformed to a delta as shown on the right thus making the circuit solvable with series/parallel techniques.*

Equations 215.2 are used to transform three resistors arranged to form a T into three resistors arranged for form a delta, such as shown in Figure 215.6. An actual example is worked out in Figure 215.8.

$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$ **Equation 215.2a**

$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$ **Equation 215.2b**

$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$ **Equation 215.2c**

The previous circuit of Figure 215.6 has two of the components moved slightly so they are drawn at an angle which now reveals that the circuit can be visualized to have a delta arrangement of three resistors as shown in Figure 215.7. Note that Figure 215.6 and Figure 215.7 are the same circuit. The circuit can be solved by converting this triangle or delta arrangement of three resistors into a T arrangement of three resistors as shown on the right side of Figure 215.7 using the transformation Equations 215.3. The circuit can be analyzed using either method. An actual example is worked out in Figure 215.8.

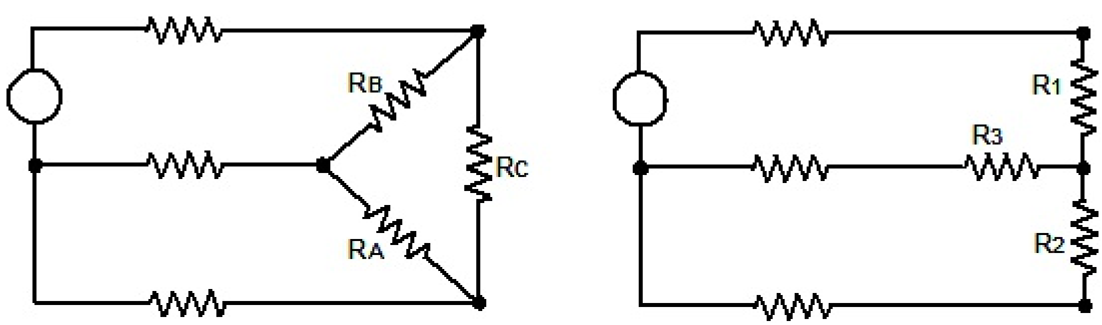

Figure 215.7 *Three resistors forming a delta within a circuit can be transformed into three resistors forming a T to make the circuit solvable using series/parallel techniques.*

$$R_1 = \frac{R_B \times R_C}{R_A + R_B + R_C}$$ **Equation 215.3a**

$$R_2 = \frac{R_C \times R_A}{R_A + R_B + R_C}$$ **Equation 215.3a**

$$R_3 = \frac{R_A \times R_B}{R_A + R_B + R_C}$$ **Equation 215.3a**

Using the Delta - T or the T - Delta Transformation: When analyzing a circuit and a point is reached where the normal series/parallel techniques cannot be applied, identify the components that are blocking progress and perform a transformation. Figure 215.8 shows the earlier circuit with actual resistor values applied. The circuit on the left is transformed to the circuit on the right. Figure 215.8 transforms a T into a delta (top) and then for the same circuit transforms a delta into a T (bottom). Once the circuit has been transformed the next step is to reduce the circuit and determine the total resistance (R_T) of the circuit and the total current (I_T). Some circuits are so complex that several successive transformations are necessary. This is a tedious process and at this point it is best to employ the help of a network solving computer program. For the example of Figure 215.8, the total resistance (R_T) by either method is 4.68 Ohm and the total current (I_T) is 25.64 ampere.

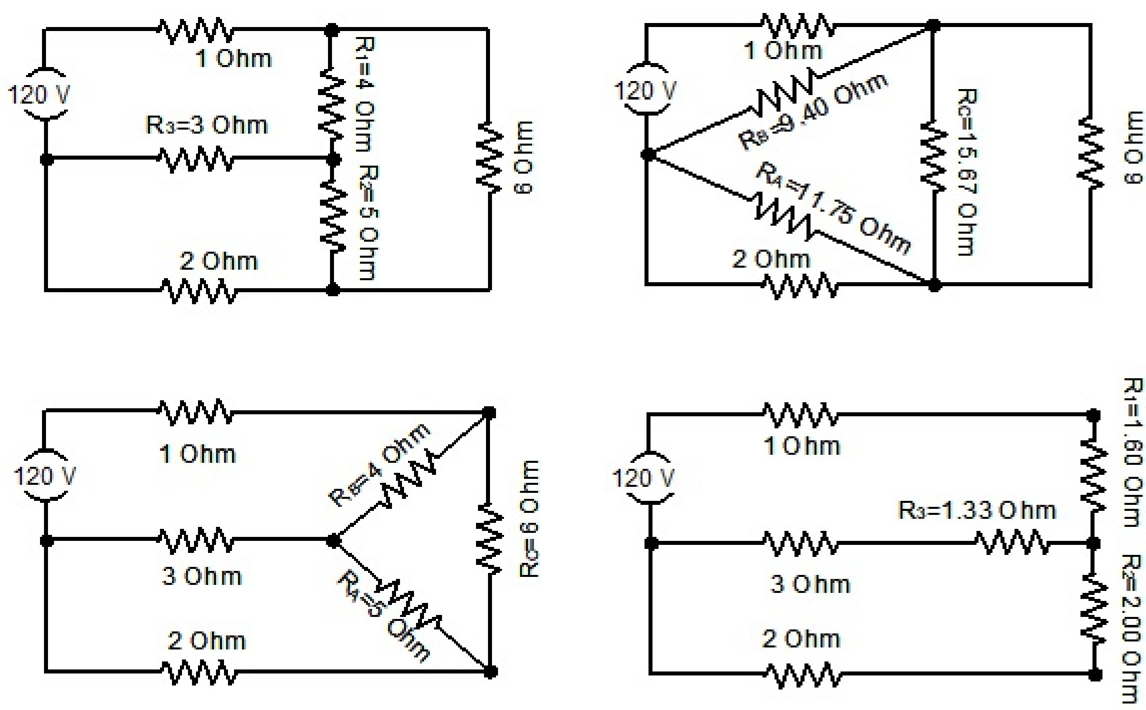

Figure 215.8 *This is an actual example of a T to delta transformation and then a delta to T transformation for the same circuit using the Equations 215.2 and Equations 215.3.*

Steps for Solving a Circuit Using T-Delta or Delta-T Transformation: Once the transformation has been completed a recommended procedure is to reduce the new circuit down and determine the total resistance of the circuit and the total current flowing. Also using the transformed circuit determine the current flow through each component of the circuit that was not transformed. These values will be the same in the original circuit as well as in the transformed circuit. Substitute these values back into the original circuit. There now should be enough information about the original circuit to determine any unknown quantity desired in the original circuit. The following steps explain a process that generally works well.

1. In order for the transformation to be a valid procedure, the current through and the voltage across the components of the circuit not transformed must remain unchanged for the transformed circuit.
2. If necessary, redraw the transformed circuit so that it is clear which components are in series and which are in parallel. (see Figure 215.9)
3. Next reduce the circuit with series/parallel techniques and determine the current through each untransformed component and the voltage across each untransformed component. Also determine the total resistance of the circuit and the total current of the circuit. (see Figure 215.9 and Figure 215.10)
4. On the original circuit write in all currents and voltages for the untransformed components. Be cautions of round-off error. (Values from Figure 215.10 are placed in the original circuit of Figure 215.11)
5. Using Kirchhoff's current law and voltage law, determine the current through each transformed component and the voltage drop across each transformed component. (Figure 215.11)
6. Finally use Ohm's law to determine the voltage drop across the transformed components. To verify the solution apply Kirchhoff's voltage law to each loop of the circuit and Kirchhoff's current law to each node of the circuit. If the solution checks within a very small round-off error then the solution is correct.

Example Analysis of Circuit of Figure 215.8: Consider the delta -T transformation of the bottom circuit of Figure 215.8. Step 2 above suggests redrawing the transformed circuit for easier visualization as shown in Figure 215.9. The circuit of Figure 215.9 is the same as the lower right hand circuit of Figure 215.8. This circuit can easily be reduced to determine the total circuit current (I_T) of 25.64 ampere and the total resistance (R_T) of 4.68 ohm.

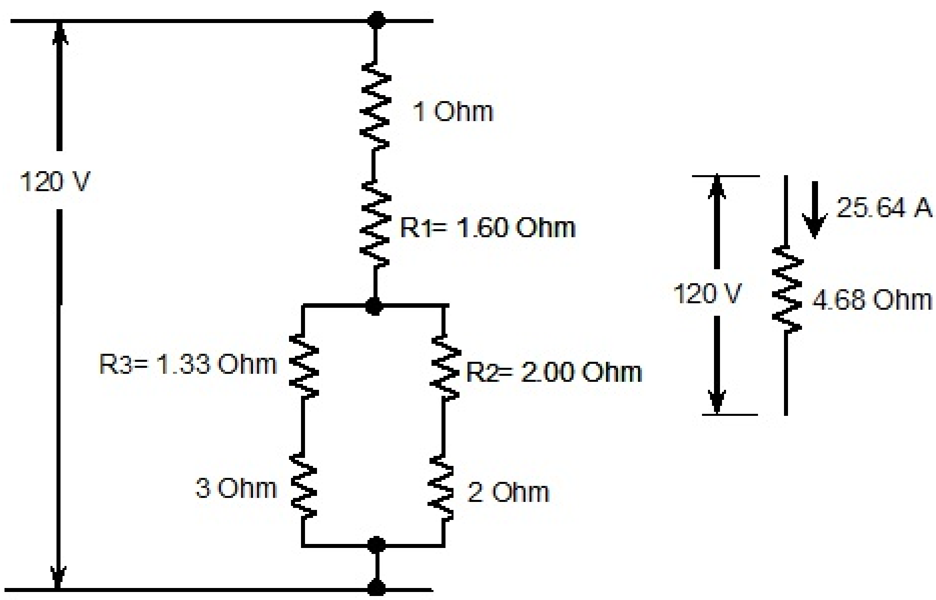

Figure 215.9 The delta -T transformed circuit of Figure 215.8 is rearranged for better visualization of the circuit.

Once the total current (25.64 ampere) is known, put the value back into the transformed circuit of Figure 215.9 and determine the current flow through and the voltage across each component of the circuit as shown in Figure 215.10.

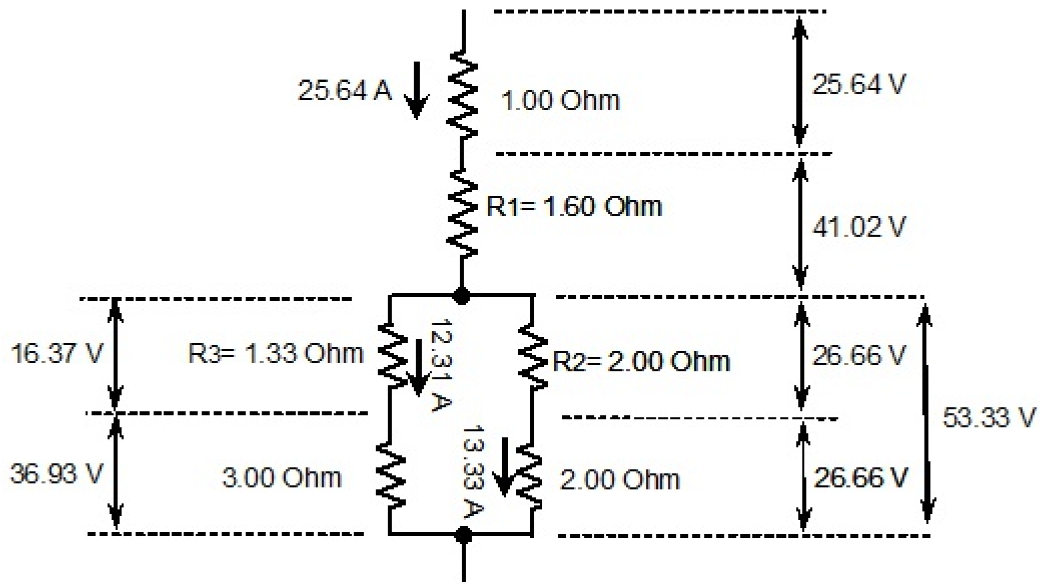

Figure 215.10 *For the delta -T transformed circuit of Figure 215.8, the current through and the voltage across each component is determined.*

The values of current through and the voltage across each component of the circuit of Figure 215.10 that was not transformed (1 ohm, 2 ohm, and 3 ohm resistors) are the same for the transformed circuit and the original untransformed circuit. Next insert these values from Figure 215.10 into the original untransformed circuit as shown in Figure 215.11.

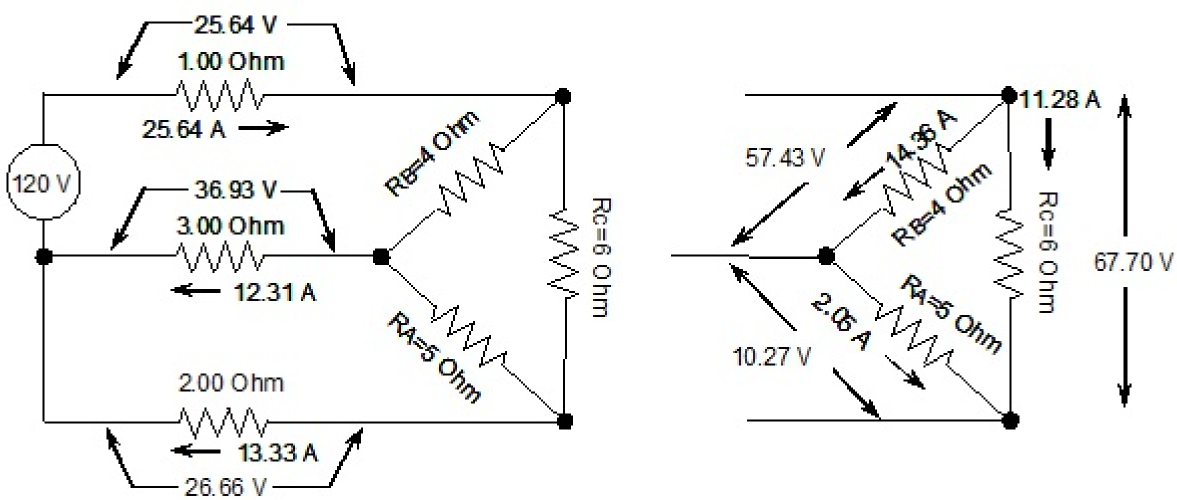

Figure 215.11 *The values of current and voltage from the transformed circuit of Figure 215.10 are placed in the original circuit.*

Now the voltages across the resistors R_A, R_B, and R_C can be determined using Kirchhoff's voltage law and the current through the resistors R_A, R_B, and R_C can be determined using Krchhoff's current law. The values of current and voltage for each component of the original circuit of Figure 215.8 (bottom left) are shown in Figure 215.11.

All of the values of voltage across and current through each resistor of the original example circuit have been determined. The solution can be verified by applying Ohm's law to each component, Kirchhoff's voltage law to each loop, and Kirchhoff's current law to each node.

Summary: Common electrical circuits can be solved to determine any quantity desired using the series/parallel reduction technique, and a transformation can be applied if necessary to overcome a roadblock in the process. Simple circuits using only resistors were analyzed in this Tech Note, but complex circuits with a reactive component can also be analyzed. When a circuit has multiple voltage sources it is usually better to analyze the circuit using other techniques such as the ones discussed in *Tech Note 230*.