Electrical Tech Note — 223

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Power in AC Circuits

Power is the rate of doing work or the rate of expending energy. The electrical unit of power is the watt. One watt is equal to one joule per second. In a dc circuit or an ac circuit where the load is a resistor, the power expended by the circuit is the product of the voltage and current. Voltage is joules per coulomb and current in amperes is coulombs per second. The product of voltage and current yields joules per second. Power expended by a dc circuit can be determined by measuring the voltage and current separately and multiplying their values. As illustrated in Figure 223.1, an ac circuit supplying only a resistive load (such as an incandescent lamp), the power in watts is equal to the product of the rms value of the voltage and current.

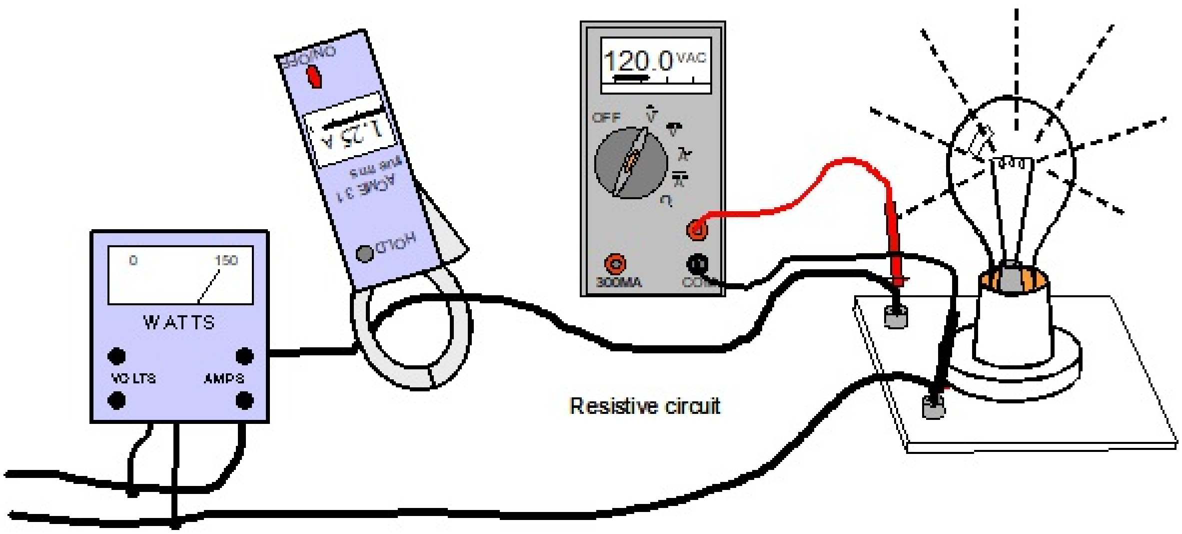

Figure 223.1 *When supplying a resistive load such as an incandescent lamp, the power in watts expended by an ac circuit is the product of the rms value of the voltage and current.*

When an ac circuit consists of a load that is made up of resistance and inductance, such as a motor, some of the current flowing in the circuit is required to supply the load and some of the current is required to build a magnetic field around the inductor coil. Energy is stored in this magnetic field at the load. If the circuit is de-energized, and the voltage taken away, the current supporting that magnetic field will stop and the magnetic field will collapse. That collapsing magnetic field will induce a voltage into the inductor wire. The energy that was stored in the magnetic field at the load is sent back to the source. This is called reactive power. For an ac circuit consisting of resistance and inductance or a circuit consisting of resistance and capacitance, some power in the circuit will be real power that is expended, and some will be reactive power that is actually returned to the source. The voltage and current in the circuit will be out of alignment by some angle theta (θ), and the product of the voltage and current (volt-amperes) will be a value greater than the actual real power in watts expended by the circuit.

In order to understand this discussion of power in ac circuits it is important to represent voltage, current, and impedance in polar and rectangular form. These quantities are explained in some detail in *Tech Note 221*. If the resistance (R), inductive reactance (X_L), and capacitive reactance (X_C) of a circuit are known, the magnitude of the impedance (Z) of that circuit can be determined using Equation 223.1.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$ **Equation 223.1**

Where reactance (X) is present in a circuit along with resistance (R), the current sine wave will be out of alignment with the voltage sine wave. The amount of this misalignment is expressed in degrees and will be some value between minus ninety degrees (-90°) and plus ninety degrees (+90°). In order to determine the angle of shift of the current from the voltage, take the inverse tangent of the reactance divided by the resistance (R) as shown in Equation 223.2. If the net reactance is positive the current will be lagging behind the voltage, and thus the angle will be negative. If the net reactance is negative, the current will be leading the voltage and the angle will be positive. The following example will shown how the magnitude of the impedance (Z_{RL}) and the shift angle are determined for an inductive circuit.

$$\theta = \tan^{-1} \frac{X}{R}$$ **Equation 223.2**

Example: A circuit, shown in Figure 223.2, is powered by a 120 volt, 60 Hz ac supply and consists of an 8. 4 ohm resistor in series with an inductor with an inductive reactance (X_L) of 10 ohms. Determine the impedance of the circuit in polar form.

Answer: First determine the magnitude of the impedance using Equation 223.1, and then the angle between the voltage and current using Equation 223.2.

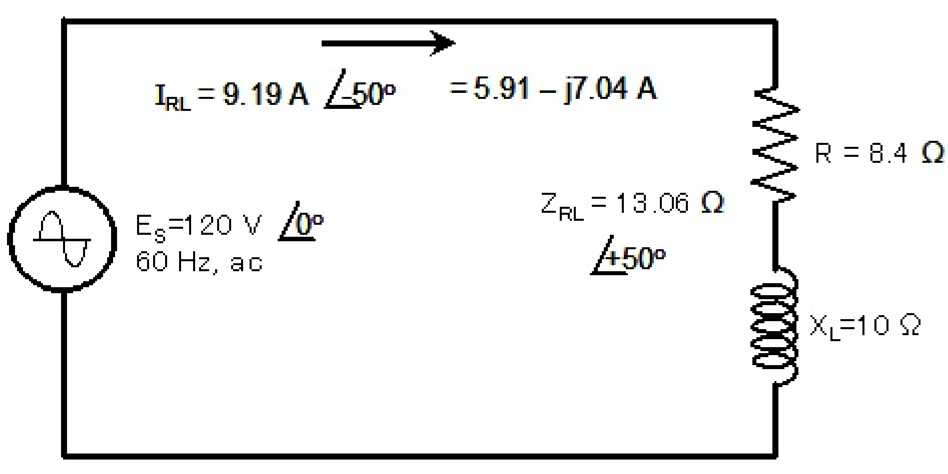

Figure 223.2 *A circuit powered by a 120 volt, 60 Hz, ac supply consists of a 8.4 ohm resistance in series with an inductive reactance of 10 ohms has an impedance of 13.08 ohm ∠+50°. The current is shown both in polar and rectangular form.*

$$Z = \sqrt{8.4^2 + 10^2} = 13.06\Omega$$

$$\theta = \tan^{-1}\frac{10}{8.4} = +50^{o}$$

Refer to *Tech Note 221* for a review of multiplication and division of quantities such as voltage, current, and impedance that are expressed in polar form. The impedance of the circuit shown in Figure 223.2 was determined in polar form as Z = 13.06 ohm $\angle$+50°. The voltage of the circuit was given as 120.00 volts $\angle$0°. Equation 223.3 is simply Ohm's law arranged to determine current when the voltage and impedance of the circuit are known.

$$I = \frac{E}{Z}$$ **Equation 223.3**

Example continued: Next divide the circuit voltage by the impedance to determine the current using Equation 223.3. Divide the magnitude of the voltage by the magnitude of the impedance to obtain the magnitude of the current. To determine the angle the current is shifted from the voltage, change the sign of the angle in the denominator (Z) and add it to the angle in the numerator. The current for the circuit of Figure 223.2 will be 9.19 amperes with an angle of minus fifty degrees (-50°). Both the voltage and current of the circuit of Figure 223.2 are represented in polar form and as sine waves in Figure 223.3. For this inductive circuit, the current is lagging behind the voltage by 50°.

$$I = \frac{120\angle 0^\circ \text{ V}}{13.06\angle +50^\circ\ \Omega} = 9.19\angle -50^\circ \text{ A}$$

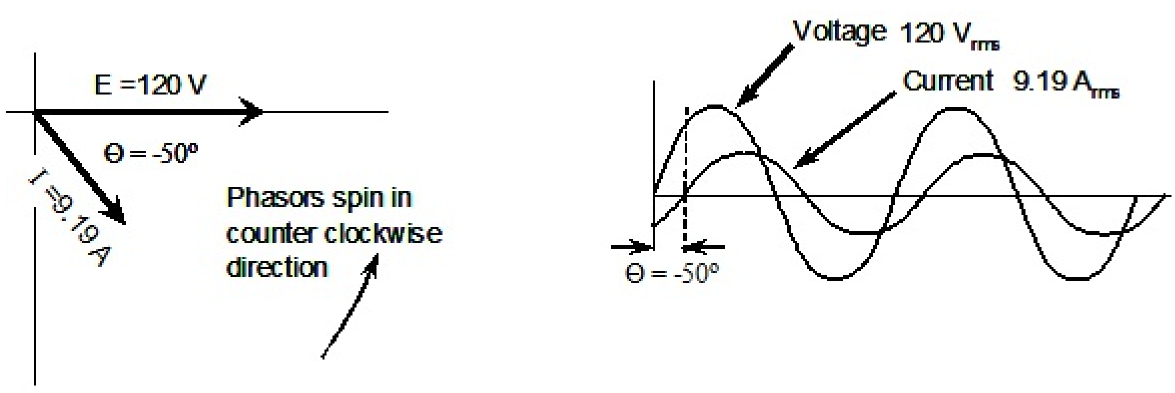

Figure 223.3 *The current for the inductive circuit of Figure 223.2 is lagging behind the voltage by an angle of 50°.*

Effect of Inductance and Capacitance on Circuit Current: When a load supplied by an ac circuit is pure resistance, there is no significant magnetic or electrostatic field present to influence the relationship of circuit voltage and current. The current and voltage of the ac circuit will be sine waves that are in alignment with each other. The current and voltage will be zero at the same time, and they will reach a maximum at the same time as shown in Figure 223.4. If the current and voltage sine waves are multiplied together another sine wave pattern will be created with all values positive. The value of power at any instant in time will be the point on the volt-ampere (apparent power) curve. The energy expended by the circuit will be the net positive area under the volt-ampere curve. In the case of a resistive load supplied by an ac source, all of the area under the volt-ampere curve is positive. Note in Figure 223.4 the current and voltage are in perfect alignment and the angle theta (θ) is zero. If the current and voltage were represented in polar form, such as in Figure 223.3, the angle θ would be zero and the current and voltage would point in the same direction.

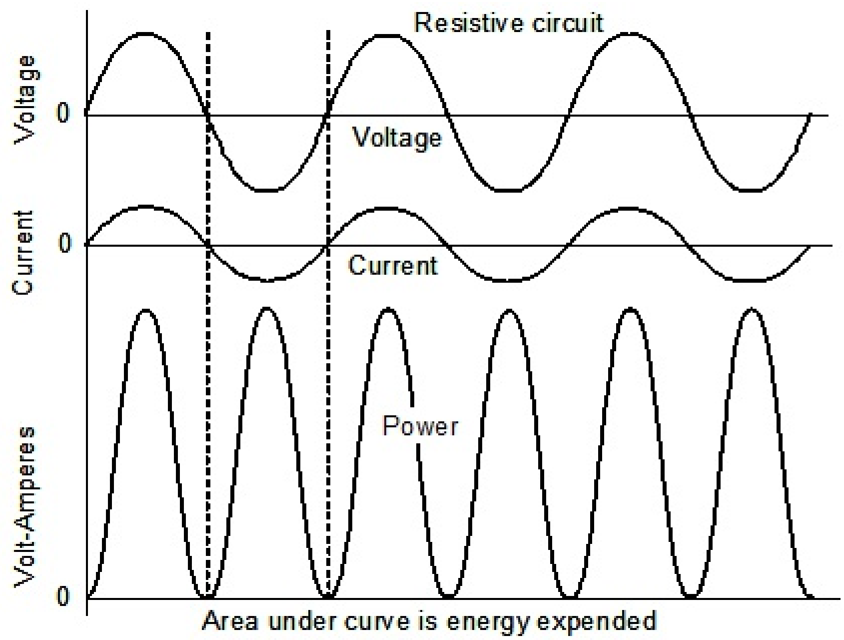

Figure 223.4 *In the case of an ac circuit with a purely resistive load, the voltage and current will be in perfect alignment, the shift angle θ will be zero, and the product of voltage and current will be all positive.*

When an ac circuit supplies a load that consists of inductance or capacitance in addition to resistance, the result will be a shift in the current sine wave with respect to the voltage sine wave by some angle θ. Figure 223.5 represents the voltage and current in an ac circuit with an inductive load where the current is lagging behind the voltage by an angle θ of 50°. This is the same off-set angle as in the previous example. Note in Figure 223.5 that when the voltage and current are not in perfect alignment, there will be times when the voltage and current have opposite signs thus resulting in negative volt-amperes. The real power expended in the circuit is the net positive volt-amperes. When the voltage and current sine waves are out-of-alignment, the apparent power is greater than the real power. Real energy expended in Figure 223.5 is the net positive area between the volt-ampere curve and the zero line.

In a circuit with an abundance of inductance along with resistance, the current sine wave will lag behind the voltage sine wave. If there is an abundance of capacitance in the circuit plus the resistance, the current sine wave will lead the voltage sine wave.

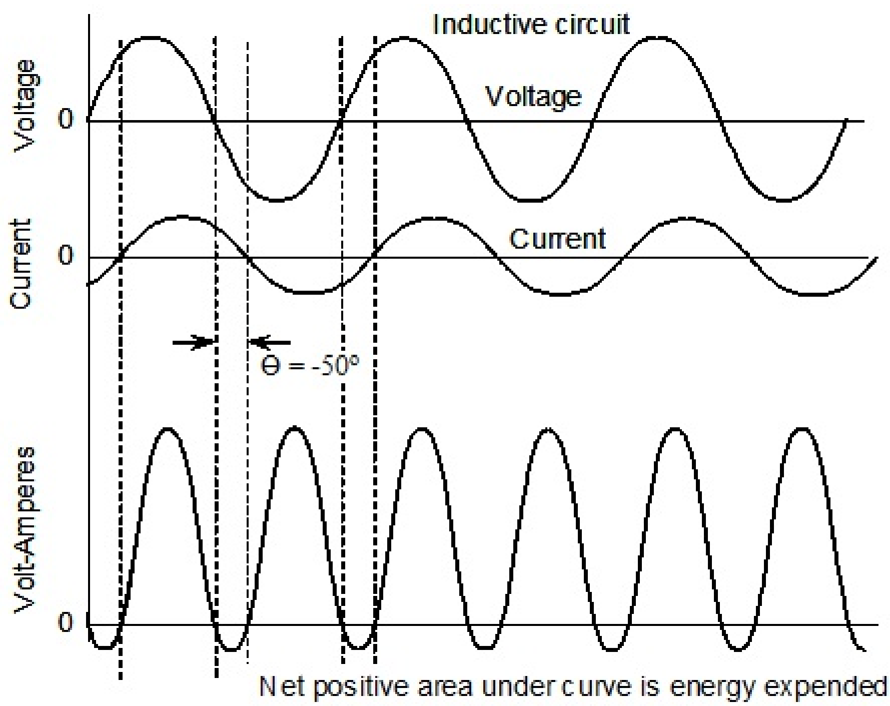

Figure 223.5 *In the case of an ac circuit where the load consists of inductance as well as resistance, the current will lag behind the voltage by an angle θ, and some of the volt-amperes will be negative. Real power expended in the circuit is the net positive volt-amperes.*

Apparent power of an ac circuit is in volt-amperes and is simply the product of the voltage and current. The apparent power forms the hypotenuse of a right triangle where the real power is the horizontal leg and reactive power is the vertical leg. Figure 223.6 is a diagram showing the relationship of real power, apparent power, and reactive power. Real power is the quantity that is expended by the circuit. It is energy that has been used doing work and producing heat or light. Once used this energy is gone. In an inductor energy is stored in a magnetic field, and that energy is returned to the circuit when the current that produces it stops. Likewise in a capacitor, energy is stored in the dielectric in the form of an electric field. That energy is returned to the circuit when the voltage that produces it stops. These concepts are discussed in detail in *Tech Note 512* and *Tech Note 513*. This energy that is stored by an inductor or a capacitor is called reactive power. This reactive power requires current in the circuit, but it does no work. In Figure 223.6 the reactive power is represented by the vertical component of the power diagram. The *apparent power* is the hypotenuse of the right triangle created by the real power on the horizontal side and the reactive power on the vertical side. The apparent power is the combined effect of the real power and reactive power. *It is simply the product of the voltage and total current* as shown in Equation 223.4. The *units of apparent power are volt-amperes (VA).* The values of apparent power, real power, and reactive power for the previous example are also shown in Figure 223.6. How these values are determined is explained in the following discussion.

Apparent Power = Voltage X Current **Equation 223.4**

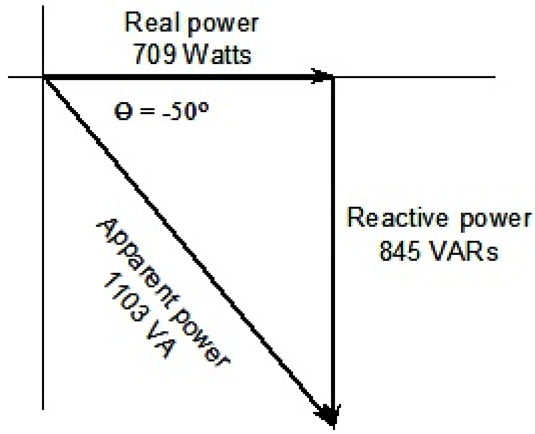

Figure 223.6 *Apparent power is the product of voltage and current in an ac circuit and it forms the hypotenuse of a right triangle on a power diagram with an angle θ with respect to the horizontal axis. The horizontal leg of the triangle is the real power expended in the circuit and the vertical leg is the reactive power associated with the magnetic or electrostatic field of the load.*

Since the power diagram of Figure 223.6 is a right triangle, trigonometry can by used to determine the real power and the reactive power of the circuit given the apparent power and the angle (θ). The real power is the product of voltage and current (VA) times the cosine of the angle θ as shown in Equation 223.5. The reactive power is the product of voltage and current (VA) times the sine of the angle θ as shown in Equation 225.6. *Real power is in units of watts and reactive power is in units of VARs (volt-amperes reactive).*

$$\text{Real Power} = \text{VA} \times \cos\theta$$ **Equation 223.5**

$$\text{Reactive Power} = \text{VA} \times \sin\theta$$ **Equation 223.6**

Example continued: Determine the apparent power, real power, and reactive power for the example of Figure 223.2. To determine apparent power used Equation 223.4. Real power is determined using Equation 223.5, and reactive power is determined using Equation 223.6. These values are displayed in Figure 223.6.

$$\text{Apparent Power} = 120\angle 0^\circ\ \text{V} \times 9.19\angle -50^\circ\ \text{A} = 1103\angle -50^\circ\ \text{VA}$$

$$\text{Real Power} = 1103\ \text{VA} \times \cos(-50^\circ) = 709\ \text{watts}$$

$$\text{Reactive Power} = 1103\ \text{VA} \times \sin(-50^\circ) = 845\ \text{VARs}$$

Power Formulas: The commonly known formula for determining power in an ac circuit is shown in Equation 223.7. This is for a single-phase circuit. *Power is the product of the voltage, current, and something called the power factor.* This equation is derived from Figure 223.6 and *power factor is the cosine of the angle θ*. *Power factor is 1.0* when the angle is zero and is zero when the angle is 90°. Circuits discussed thus far are single-phase circuits. This means the ac source produces only one sine wave to power the circuit. Frequently circuits for which power must be determined are 3-phase circuits where there are three conductors from the source with three different sine wave combinations between the pairs of conductors. Each of these sine waves are spaced 120° off-set from each other. Typically the voltage between the three conductors will be nearly identical. The current for a specific 3-phase load will be approximately the same in each conductor. For such a load, one current and one voltage are used in the power formula except the 3-phase power formula has the *voltage, current, and power factor multiplied by the square root of three,* which is the number 1.73. To determine power for a 3-phase circuit use Equation 223.8.

Single-Phase Power:

$$\text{Power} = \text{Voltage} \times \text{Current} \times \text{power factor} \qquad \textbf{Equation 223.7}$$

3-Phase Power:

$$\text{Power} = 1.73 \times \text{Voltage} \times \text{Current} \times \text{power factor} \qquad \textbf{Equation 223.8}$$

Power Factor: Examining the previous example and Figure 223.6 and it can be seen that the real power of an ac circuit is the voltage times the current times the cosine of the angle θ. For *an ac circuit where the load is pure resistance, the angle θ will be 0° and Cos 0° is 1.0.* For the opposite extreme, the angle θ may be either +90° (capacitive) or it may be −90° (inductive). In either case Cos +90° or Cos −90° is zero. This means no matter how much current is flowing, there is no real power being expended by the circuit. The energy stored in the magnetic field, or electrostatic field of the load is sent back to the source when the voltage is removed. The values of Cos θ then vary from 0.0 to 1.0 depending upon the ratio of resistance to reactance in the circuit. Cos θ in the power formula is called the power factor. Power factor can be measured directly or it can be determined by measuring the circuit voltage and current, and measuring the power draw of the circuit in watts. Power factor can be calculated from Equation 223.7 or Equation 223.8 and is the wattage divided by the volt-amperes as shown by Equation 223.9. In the 3-phase case the wattage is divided by 1.73 times the volt-amperes as shown by Equation 223.10.

Single-Phase Circuit Power Factor:

$$\text{power factor} = \frac{\text{Watts}}{\text{Volt X Amperes}} \qquad \textbf{Equation 223.9}$$

3-Phase Circuit Power Factor:

$$\text{power factor} = \frac{\text{Watts}}{1.73 \text{ X Volts X Amperes}} \qquad \textbf{Equation 223.10}$$

Analysis of Circuits With Reactive Power: The real power expended by the circuit is the result of the circuit current flowing through the resistance of the circuit. The reactive power of the circuit is the circuit current flowing through the inductive or capacitive reactance of the circuit. The example of Figure 223.2 gave the value of the resistance and inductive reactance of the circuit. The apparent power of a circuit can be determined by multiplying the impedance of the circuit and the square of the current as shown in Figure 223.11. Real power expended can then be determined by multiplying the resistance of the circuit and the square of the current as shown by Equation 223.12. Likewise, the reactive power is the product of the circuit reactance and the square of the current as shown by Equation 223.13. For the previous example, the values of apparent power, real power, and reactive power are determined by multiplying the square of the current and either the total impedance, resistance, and reactance of the circuit.

$P_{APPARENT} = I^2 \times Z$ **Equation 223.11**

Apparent Power = $(9.19\ A)^2 \times 13.06\ \Omega = 1103$ watts

$P_{REAL} = I^2 \times R$ **Equation 223.12**

Real Power = $(9.19\ A)^2 \times 8.4\ \Omega = 709$ watts

$P_{REACTIVE} = I^2 \times X_L$ **Equation 223.13**

Reactive Power = $(9.19\ A)^2 \times 10\ \Omega = 845$ VARs

The apparent power of the circuit was shown by Equation 223.4 to be the product of the current of the circuit and the supply voltage. The real power and reactive power of a circuit can be obtained as the product of resistance and reactance and the voltage across each of these components. First determine the voltage across the resistance by multiplying the circuit current and circuit resistance as shown by Equation 223.14, and the voltage across the reactance of the circuit by multiplying the circuit current and the reactance using Equation 223.15. There is 77.2 volts across the resistor of the example circuit, and 91.9 volts across the inductor (in this case inductive reactance). The product of each of these voltages and the circuit current will give the real and reactive power as shown by Equation 223.16 and Equation 223.17. These turn out to be the same values shown in Figure 223.6.

$E_R = I \times R$ **Equation 223.14**

Voltage Across Resistor= $9.19\ A \times 8.4\ \Omega = 77.2\ V$

$E_L = I \times X_L$ **Equation 223.15**

Voltage Across Inductance = $9.19\ A \times 10\ \Omega = 91.9\ V$

$P_{REAL} = I \times E_R$ **Equation 223.16**

Real Power = $9.19\ A \times 77.2\ V = 709$ watts

$P_{REACTIVE} = I \times E_R$ **Equation 223.17**

Reactive Power = $9.19\ A \times 91.9\ V = 845$ VARs

The purpose of showing multiple ways of determining the same quantities for a circuit is to shown that the basic formulas for analyzing a simple circuit with only resistance also apply to more complex ac circuits containing resistance and reactance. The circuit of Figure 223.2 might be a representation of the windings of an electric motor. The circuit shows a separate resistor and inductor, but in reality these are not separate components in a device. The coil of wire that creates the inductive reactance also has significant resistance and the two do not exist as separate components. However, the value of resistance and the value of inductive reactance can be determined and represented as though they were separate components as shown in Figure 223.2. From these separate values the real and reactive power can be determined as well as the voltage across them if they did exist as separate components.

Significance of Power Factor: An ac circuit that operates with a small shift between the current and voltage sine waves has an apparent power with a lower magnitude than a circuit with a large angle θ. This is illustrated in Figure 223.7 for a load with a large and a small shift between the voltage and the current. Since the circuit voltage remains constant, the load with the greater angle θ will have a higher current for the same real power output. Electrical system conductors and equipment must be sized to carry the current of the load, so it is important to minimize the current required to supply a particular load by keeping the power factor high (small angle) whenever practical. The current drawn by a load can be determined by dividing the power in watts by the circuit voltage and power factor as shown in Equation 223.18. This equation is for a single-phase load. For a 3-phase load it is necessary to also divide by 1.73. The inductive reactance of common ac loads such a induction motors can be offset by the installation of a capacitor connected in parallel with the load as shown in Figure 223.8. The capacitor is installed at the load which then changes the shift angle between the voltage sine wave and the current sine wave of the circuit. By reducing this angle the power factor is increased and the current drawn by the load on the circuit conductors as shown in the right hand diagram of Figure 223.7 is reduced.

$$\text{Current} = \frac{\text{Power in watts}}{\text{Voltage} \times \text{power factor}}$$ **Equation223.18**

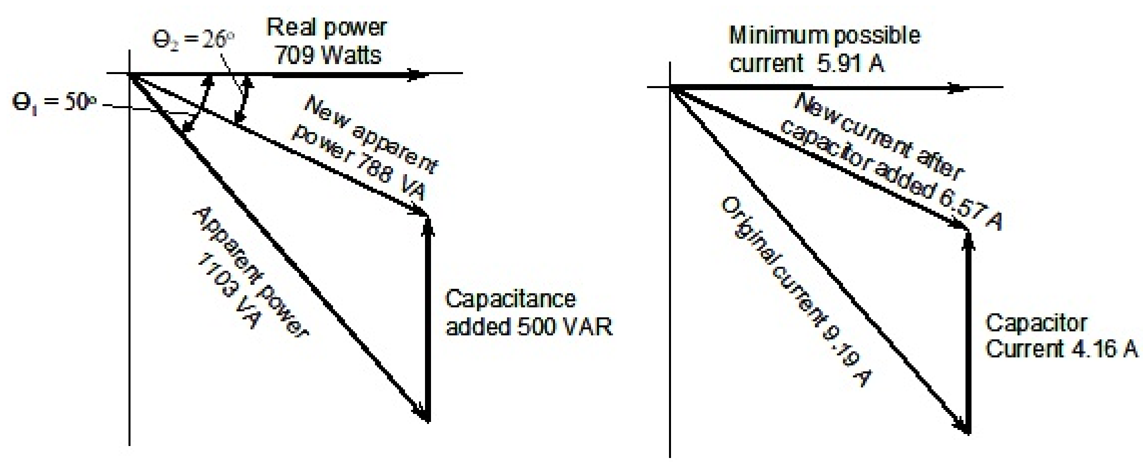

Figure 223.7 *Even though the real power in watts are the same, a load with a low power factor (large shift angle θ) has a higher apparent power and circuit current than the same load with a low power factor.*

Correcting Power Factor: Frequently power factor can be maintained at a high level by being aware of the consequences of low power factor and choosing equipment with a high power factor when such choices are available. Typical loads tend to be inductive in nature which results in the current lagging behind the voltage. The most common way to increase the power factor and reduce the shift angle between the current and the voltage of an ac circuit is to install capacitance in parallel with the load as shown in Figure 223.8. Since typical ac electrical systems operate at 60 Hz, it is customary to rate power factor correcting capacitors in VARs or kVARs rather than microfarads µf. Figure 223.8 is the same circuit as Figure 223.2 except a 500 VAR, 60 Hz capacitor has been installed in parallel with the RL branch of the circuit to correct the power factor. The effect the capacitor has on the circuit is diagramed in Figure 223.7. The capacitive reactance offsets some of the inductive reactance of the circuit. The real power in watts remains unchanged, but the apparent power is greatly reduced. As can be seen in both Figure 223.7 and Figure 223.8, the circuit current is reduced from 9.19 amperes to 6.57 amperes by the addition of this capacitor in parallel with the inductive load.

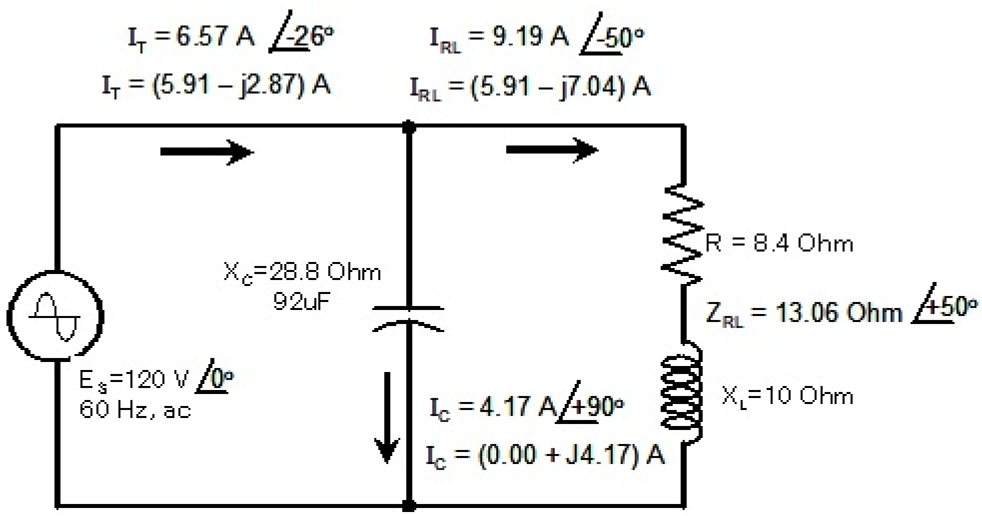

Figure 223.8 *A 500 VAR, 60 Hz capacitor is installed in parallel with the RL branch of the circuit to increase the power factor of the circuit.*

Refer to Figure 223.7 and note that real power is the horizontal leg of a right triangle, and the reactive power (VARs) is the vertical leg of the triangle. Correcting power factor involves reducing the offset angle θ usually not to zero, but to within 25° of zero. This corresponds to a power factor of 0.9 or higher. Reactive power divided by the real power is the tangent of the angle θ. This is the basis for Equation 223.19 used to determine the number of VARs or kVARs needed to correct the power factor to some desired level. For commercial and industrial buildings the approximate load in watts or kilowatts is known as well as the power factor. Meters can be installed to measure power and power factor. A target power factor is selected and the calculation performed to determine the capacitance needed to obtain the desired power factor correction.

$$\text{kVAR needed} = \text{Load kW} \times [\tan\theta_2 - \tan\theta_1] \qquad \textbf{Equation 223.19}$$

$$\theta_1 = \cos^{-1}(\text{original pf})$$

$$\theta_2 = \cos^{-1}(\text{desired pf})$$

Example: A building is served with 240 volt single-phase power and has a load of 48 kW with a power factor of 0.7. Determine the kVAR of capacitance needed to correct the power factor to 0.95.

Answer: First determine the angles θ_1 and θ_2, then determine the kVAR needed using

Equation 223.19. Figure 223.9 shows the effect of adding the capacitors to the load.

$$\theta_1 = Cos^{-1}(0.70) = 45.6°$$

$$\theta_2 = Cos^{-1}(0.95) = 18.2°$$

$$\text{kVAR needed} = 48\text{ kW} \times [\text{Tan } 45.6° - \text{Tan } 18.2°]$$

$$= 48\text{ kW} \times [1.21 - 0.33] = 42\text{ kVAR}$$

Selecting Power Factor Capacitors in µf: There may be times when it is desirable to determine the capacitance in microfarads (µf) needed to correct the power factor of a circuit. The key is to remember that VARs are simply reactive volt-amperes. The magnitude of the VARs desired to improve power factor with the capacitor must first be determined. For the previous example 42 kVAR (42,000 VAR) are needed to correct the power factor to 0.95. The capacitors will be connected across the 240 volts of the power entering the building, therefor, it is necessary to determine how much current will be drawn by these capacitors. Equation 223.20 can be used to determine the current draw of power factor correcting capacitors given their value in VARs.

$$\text{Capacitor Current Needed} = \frac{\text{VARs}}{\text{Circuit Voltage}}$$ **Equation 223.20**

Example continued: For the previous example of a 240 volt single-phase load it was determined that 42,000 VARs of capacitance was needed to correct the power factor to a level of 0.95. The current draw of those capacitors will be 175 amperes.

$$\text{Capacitor Current Needed} = \frac{42{,}000\text{ VARs}}{240\text{ V}} = 175\text{ A}$$

The next step in the process of determining the value of the capacitors in microfarads (µf) is to determine the capacitive reactance required to draw the necessary current. This is accomplished using Ohm's law as shown in Figure 223.21. For the previous example the value of capacitive reactance is $X_C = 1.37$ ohm.

$$X_C = \frac{\text{Circuit Voltage}}{\text{Capacitor Current Needed}}$$ **Equation 223.21**

$$X_C = \frac{240\text{ V}}{175\text{ A}} = 1.37\text{ ohm}$$

Finally the capacitance in microfarads (µf) is determined from the formula for determining capacitance that was discussed in Tech Note 512. That formula is reproduced here as Equation 223.22, and the value of capacitance required for the previous example is 1,936 µf. Refer back to the example of the circuit in Figure 223.7 and Figure 223.8. The capacitor selected was 500 VARs which in the 120 volt circuit would draw 4.17 amperes. To draw that current the reactance of the capacitor was 28.8 ohm. Placing these values into Equation

223.22 results in a value for the capacitor in Figure 223.8 of 92 μf.

$$\text{Capacitance} = \frac{1}{2 \times \pi \times f \times X_C}$$ **Equation 223.22**

$$\text{Capacitance} = \frac{1}{2 \times 3.14 \times 60\ \text{Hz} \times 1.37\ \Omega} = 0.001936\ \text{f} = 1{,}936\ \mu\text{f}$$

Conclusion: For a pure resistive circuit the voltage sine wave and the current sine wave are in perfect alignment. They cross the zero line at the same time and reach their peak values at the same time. A vector diagram is shown to the right with the vectors spinning counter-clockwise at 60 revolutions per second for ac power at 60 Hz.

For an inductive circuit notice the voltage and current sine waves are not in alignment. The current sine wave is lagging behind the voltage sine wave at some angle which can be as much as nearly 90° depending upon how much inductance is in the circuit.

For a capacitive circuit the current sine wave is leading the voltage sine wave and they are not in alignment. The current sine wave can lead the voltage sine wave as much as nearly 90° depending upon the amount of capacitance in the circuit.