Electrical Tech Note 215B

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DELTA – T CIRCUIT SOLUTIONS

Sometimes the components of an electrical circuit can be arranged such that the series and parallel reduction techniques do not work. An example is the circuit on the left of *Figure 215.1.* For the purposes of circuit analysis, the circuit can be changed to an alternate form that functions the same allowing the ability to apply the series and parallel reduction techniques to determine desired unknown values for components of the circuit. Note that the internal components of the circuit on the left of *Figure 215.1* form a letter T (R_1, R_2, and R_3). There is a transformation that can convert the three resistors forming the T into three resistors forming a triangle or delta as shown on the right of *Figure 215.1* (R_A, R_B, and R_C). The circuit on the right now can be solved using series and parallel reduction techniques. This transformation is known as the T - delta transformation and is accomplished using *Equations 215.1.*

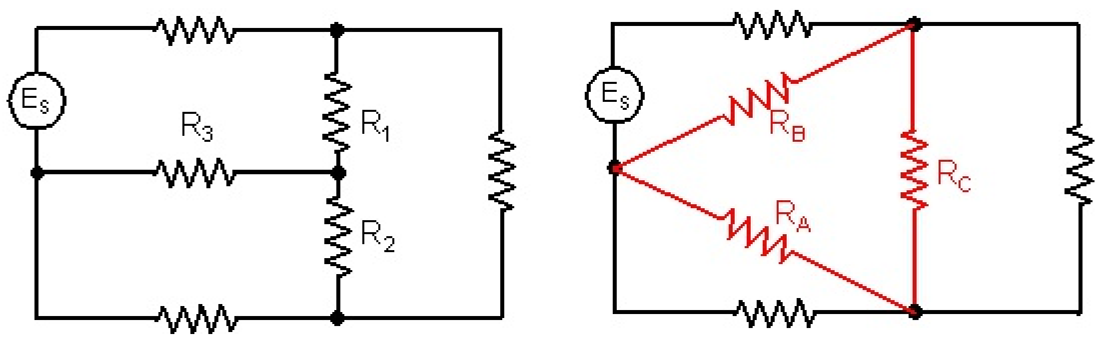

Figure 215.1 *The circuit on the left cannot be solved using typical series/parallel techniques, therefore, the components forming the T have been transformed to a delta as shown on the right thus making the circuit solvable with series/parallel techniques.*

Equations 215.1 are used to transform three resistors that form a "T" into three resistors that form a delta, such as shown in *Figure 215.1.* An example is worked out in *Figure 215.3.*

$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$ **Equation 215.1a**

$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$ **Equation 215.1b**

$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$ **Equation 215.1c**

The previous circuit of *Figure 215.1* has two of the components moved slightly so they are drawn at an angle which now reveals that the circuit can be visualized to have a delta arrangement of three resistors as shown in *Figure 215.2.* Note that *Figure 215.1* and *Figure 215.2* are the same circuit. The circuit can be solved by converting this triangle or **delta** arrangement of three resistors into a **T** arrangement of three resistors as shown on the right side of *Figure 215.2* using the transformation *Equations 215.3.* The circuit can be analyzed using either method. An actual example is worked out in *Figure 215.3.*

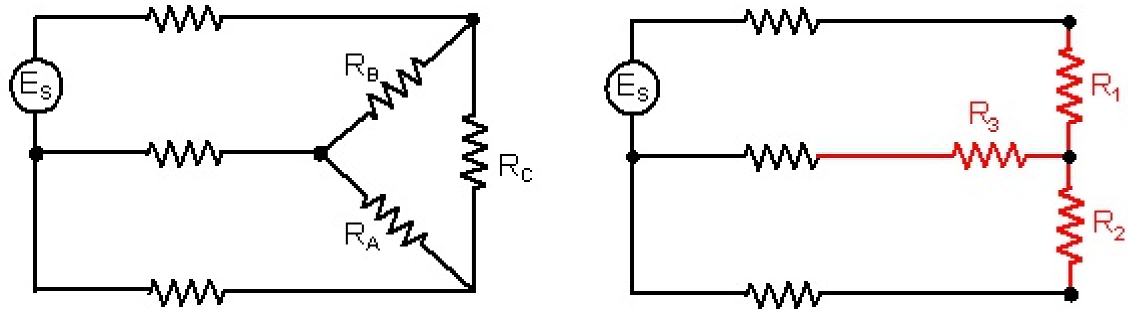

Figure 215.2 *Three resistors forming a delta within a circuit can be transformed into three resistors forming a T to make the circuit solvable using series/parallel techniques.*

$$R_1 = \frac{R_B \times R_C}{R_A + R_B + R_C}$$ **Equation 215.2a**

$$R_2 = \frac{R_C \times R_A}{R_A + R_B + R_C}$$ **Equation 215.2a**

$$R_3 = \frac{R_A \times R_B}{R_A + R_B + R_C}$$ **Equation 215.3a**

Using the Delta - T or the T - Delta Transformation: When analyzing a circuit and a point is reached where the normal series/parallel techniques cannot be applied, identify the components that are blocking progress and perform a transformation. *Figure 215.3* shows the earlier circuit with actual resistor values applied. The circuit on the left is transformed to the circuit on the right. *Figure 215.3* transforms a **T** into a **delta** (top) and then for the same circuit transforms a **delta** into a **T** (bottom). Once the circuit has been transformed the next step is to reduce the circuit and determine the total resistance (R_T) of the circuit and the total current (I_T). Some circuits are so complex that several successive transformations are necessary. This is a tedious process and at this point it is best to employ the help of a network solving computer program. For the example of *Figure 215.3*, the total resistance (R_T) by either method is 4.68 ohm and the total current (I_T) is 25.64 ampere.

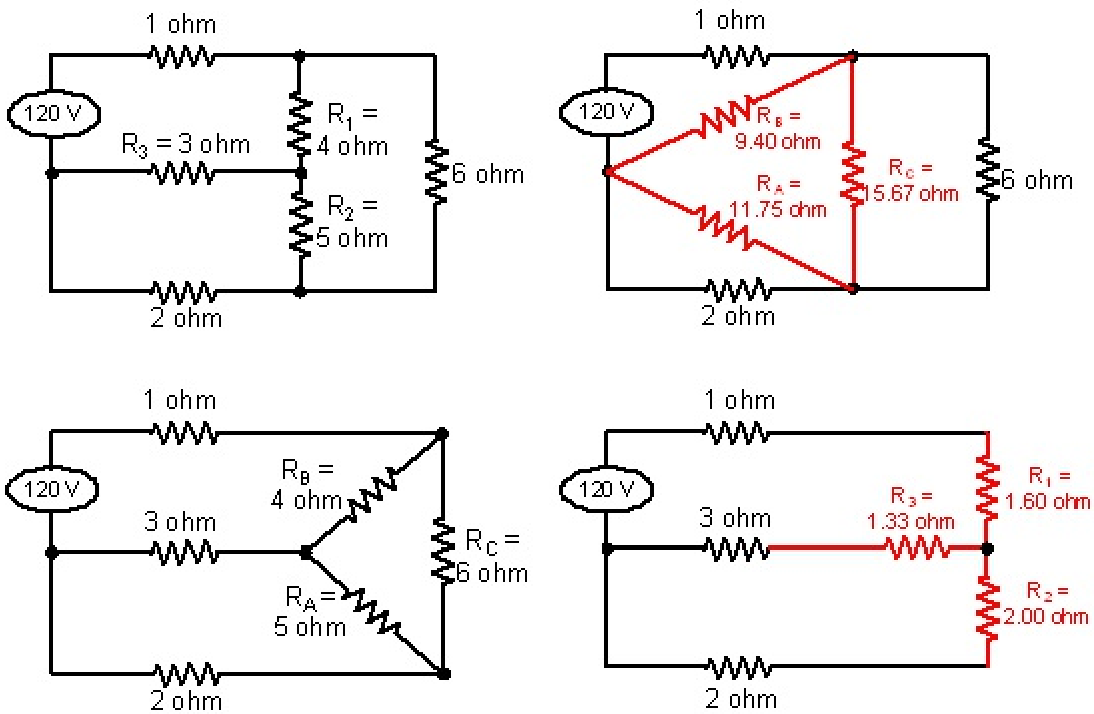

Figure 215.3 *This is an actual example of a T to delta transformation and then a delta to T transformation for the same circuit using the Equations 215.1 and Equations 215.2.*

Steps for Solving a Circuit Using T-Delta or Delta-T Transformation: Once the transformation has been completed a recommended procedure is to reduce the new circuit down and determine the total resistance of the circuit and the total current flowing. Also using the transformed circuit determine the current flow through each component of the circuit that was not transformed. These values will be the same in the original circuit as well as in the transformed circuit. Substitute these values back into the original circuit. There now should be enough information about the original circuit to determine any unknown quantity desired in the original circuit. The following steps explain a process that generally works well.

1. In order for the transformation to be a valid procedure, the current through and the voltage across the components of the circuit not transformed must remain unchanged for the transformed circuit.
2. If necessary, redraw the transformed circuit so that it is clear which components are in series and which are in parallel. (see *Figure 215.4*)
3. Next reduce the circuit with series/parallel techniques and determine the current through each untransformed component and the voltage across each untransformed component. Also determine the total resistance of the circuit and the total current of the circuit. (see *Figure 215.4* and *Figure 215.5*)
4. On the original circuit write in all currents and voltages for the untransformed components. Be cautions of round-off error. (Values from *Figure 215.5* are placed in the original circuit of *Figure 215.6*)
5. Using Kirchhoff's current law and voltage law, determine the current through each transformed component and the voltage drop across each transformed component. (*Figure 215.6*)

6. Finally use Ohm's law to determine the voltage drop across the transformed components. To verify the solution apply Kirchhoff's voltage law to each loop of the circuit and Kirchhoff's current law to each node of the circuit. If the solution checks within a very small round-off error then the solution is correct.

Example Analysis of Circuit of *Figure 215.3*: Consider the delta -T transformation of the bottom circuit of *Figure 215.3.* Step 2 above suggests redrawing the transformed circuit for easier visualization as shown in *Figure 215.4.* The circuit of *Figure 215.4* is the same as the lower right hand circuit of *Figure 215.3.* This circuit can easily be reduced to determine the total circuit current (I_T) of 25.64 ampere and the total resistance (R_T) of 4.68 ohm.

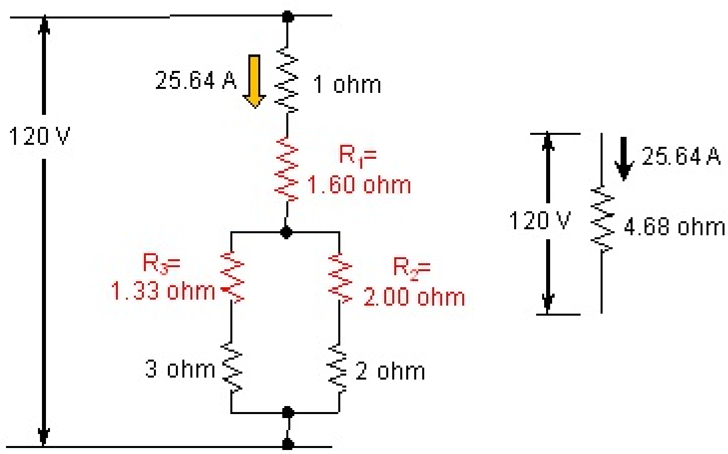

Figure 215.4 *The delta -T transformed circuit of Figure 215.3 is rearranged for better visualization of the circuit.*

Once the total current (25.64 ampere) is known, put the value back into the transformed circuit of *Figure 215.4* and determine the current flow through each component of the circuit as shown in *Figure 215.5.* To determine the current through each branch of the parallel portion of the circuit use the current divider equation found in *Tech Note 213.*

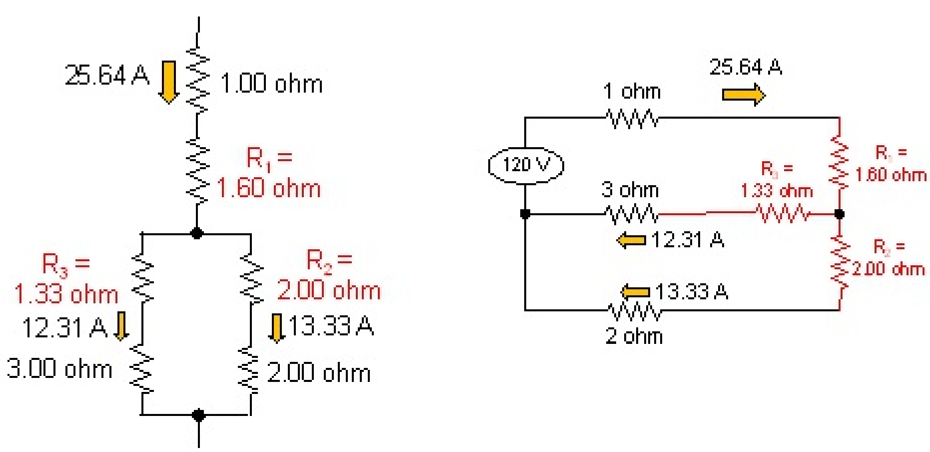

Figure 215 *Once the current through each resistor has been determined put those values into the transformed circuit next to the original resistors that were not transformed.*

The final step is to put the current values for the original resistors back into the original circuit as shown in *Figure 214.6.* This is enough information to determine the current resistors R_A, R_B, and R_C. Use Kirchhoff current law and voltage law. All of the values of voltage across and current through each resistor of the original example circuit have been determined. The solution can be verified by applying Ohm's law to each component, Kirchhoff's voltage law to each loop, and Kirchhoff's current law to each node.

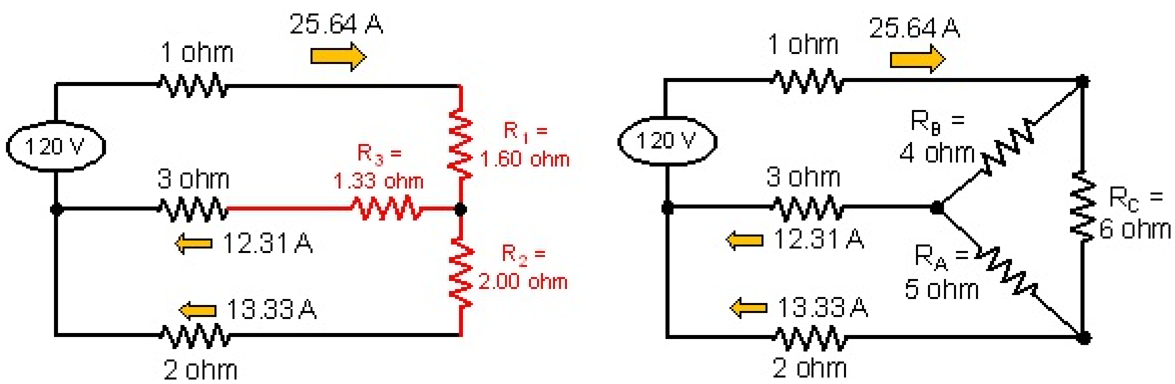

Figure 215.6 *The values of current and voltage from the original transformed circuit can be determined using Kirchhoff current law and voltage law.*

Summary: Common electrical circuits can be solved to determine any quantity desired using the series/parallel reduction technique, and a transformation can be applied if necessary to overcome a roadblock in the process. Simple circuits using only resistors were analyzed in this *Tech Note*, but complex circuits with a reactive component can also be analyzed. When a circuit has multiple voltage sources it is usually better to analyze the circuit using other techniques such as the ones discussed in *Tech Note 230*.

Solution: R_A ***= 2.05A,*** R_B ***= 14.36 A, and*** R_C ***= 11.28 A***