Electrical Tech Note — 230B

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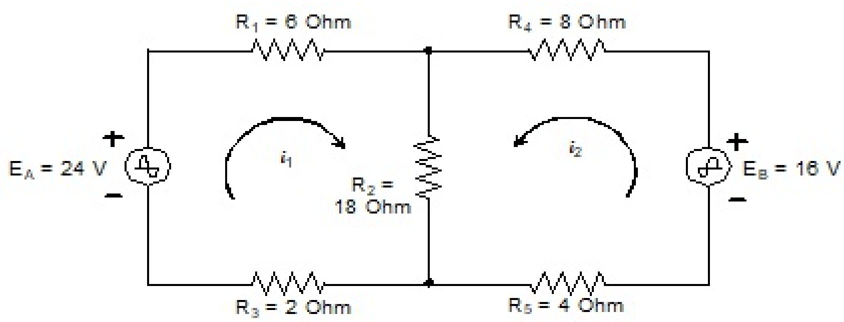
Solving Loop Equations

Here is a method of solving electrical circuits with multiple voltage sources using Kirchhoff voltage loop equations. Forming a matrix equation and solving using Cramer's Rule. There are only two loops in this network and only a 2 by 2 determinant is required for the solution. To write the loop equations the current arrows can be drawn by any method, but be very careful to get the sign correct for each value. It is suggested the arrows be drawn based on the polarity of the voltage source for each loop. Later in this Tech Note the current arrows will be drawn in other directions to show how it is important to keep track of coefficient signs.



$6i_1 + 18(i_1 + i_2) + 2i_1 = 24$

$(6 + 18 + 2)\ i_1 + 18\ i_2 = 24$

$8i_2 + 18(i_1 + i_2) + 4i_2 = 16$

$18\ i_1 + (8 + 18 + 4)\ i_2 + = 16$

Left loop equation

$$26i_1 + 18i_2 = 24$$

Right loop equation

$$18i_1 + 30i_2 = 16$$

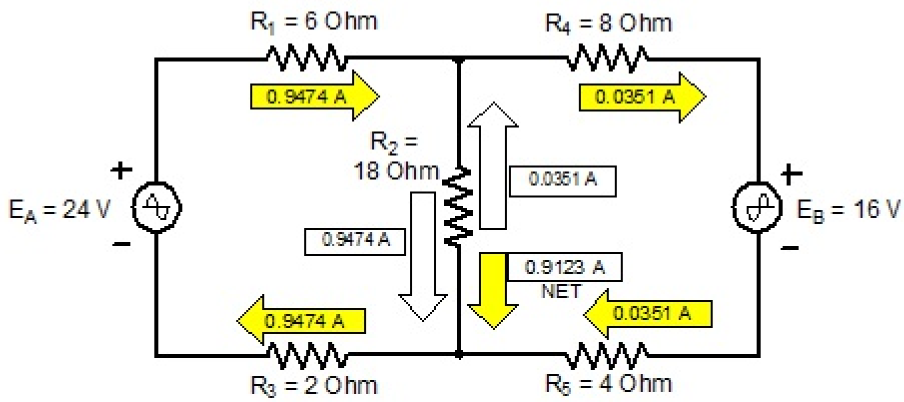
$$\begin{bmatrix} 26 & 18 \\ 18 & 30 \end{bmatrix} \bullet \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 24 \\ 16 \end{bmatrix}$$

Cramer's Rule:

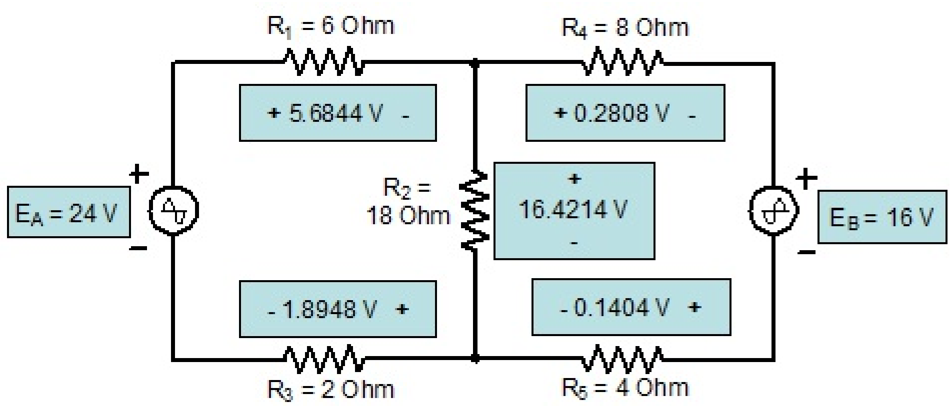
$$i_1 = \frac{\begin{vmatrix} 24 & 18 \\ 16 & 30 \end{vmatrix}}{\begin{vmatrix} 26 & 18 \\ 18 & 30 \end{vmatrix}} = \frac{720 - 288}{780 - 324} = \frac{432\ V}{456\ ohm} = 0.9474\ A$$

$$i_2 = \frac{\begin{vmatrix} 26 & 24 \\ 18 & 16 \end{vmatrix}}{\begin{vmatrix} 26 & 18 \\ 18 & 30 \end{vmatrix}} = -\ 0.0351\ A$$

The following circuit diagram shows the calculated currents for the two loops placed on the circuit diagram. The minus sign for i_2 indicates that the loop current arrow on the previous page was not the direction of current flow in the right hand loop. Note that the current i_2 calculated for that loop is shown in the correct direction in the following circuit diagram. The actual current flowing through the resistor R_2 for this diagram is the net difference of the loop currents.



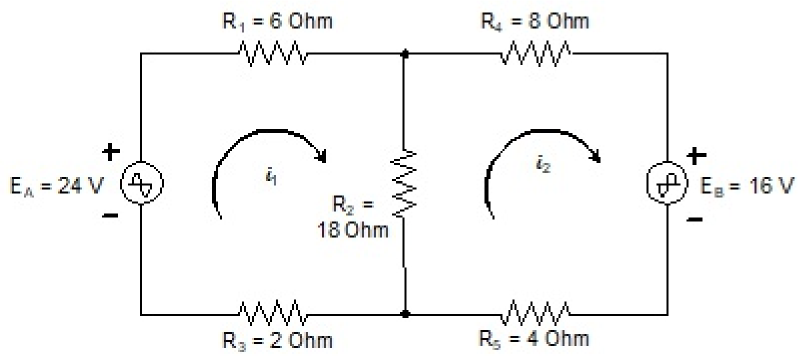
Since this method is based on Kirchhoff's voltage low a good way to verify that the solution for loop currents is correct is to multiply each value of resistor by it's current to obtain the voltage drop across the resistor. Than sum the resistor voltage drop for each loop to determine is the loop voltage values are equal to the source voltage of each loop. Be careful since the voltages across the resistors may subtract rather than add. The following circuit diagram shows the voltage across each resistor which for each loop add to the value of the source voltage with only a small rounding error. Notice that for this solution rounding was only to four decimals. If you do not carry enough decimal places it is highly likely an inaccurate solution may be the result.



Left Loop Voltages
+5.6844 V +16.4214 V +1.8948 V
24.0006 V

Right Loop Voltages
-0.2808 V +16.4214 V -0.1404 V
16.0002 V

It does not matter in which direction the loop currents are drawn. A correct solution can be obtained, but it is essential that the sign is correct for each element in the equations. The following circuit diagram is exactly the same as the one on page 1 except both loop currents have been drawn in a clockwise direction. If solved correctly the loop currents i_1 and i_2 will have the same magnitude.



$6i_1 + 18(i_1 - i_2) + 2i_1 = 24$

$(6 + 18 + 2)\, i_1 - 18\, i_2 = 24$

Left loop equation

$26i_1 - 18i_2 = 24$

$4i_2 + 18(i_2 - i_1) + 8i_2 = -16$

$-18\, i_1 + (4 + 18 + 8)\, i_2 + = -16$

Right loop equation

$-18i_1 + 30i_2 = -16$

$$\begin{bmatrix} 26 & -18 \\ -18 & 30 \end{bmatrix} \bullet \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 24 \\ -16 \end{bmatrix}$$

$$i_1 = \frac{\begin{vmatrix} 24 & -18 \\ -16 & 30 \end{vmatrix}}{\begin{vmatrix} 26 & -18 \\ -18 & 30 \end{vmatrix}} = 0.9474 \text{ A} \qquad i_2 = \frac{\begin{vmatrix} 26 & 24 \\ -18 & -16 \end{vmatrix}}{\begin{vmatrix} 26 & -18 \\ -18 & 30 \end{vmatrix}} = 0.0351 \text{ A}$$

Notice with this solution the direction of the loops drawn in the original circuit indicate the correct direction of the currents since both currents i_1 and i_2 are positive values.