Electrical Tech Note — 512

Biosystems &Agricultural Engineering Department
Michigan State University

Capacitors

Capacitors are frequently used components in electronic circuits. A capacitor consists of two flat metal plates facing each other and separated by an insulating material called a dielectric. If these metal plates are connected to a source of direct current, current will not flow from one plate to the other, but current will flow from the source to each metal plate to build up a charge in the dielectric. Once the voltage between the metal plates matches the dc source voltage, current flow will stop. Except for the process of charging and discharging, a capacitor can be used to block the flow of direct current. Figure 512.1 is a simple diagram of a capacitor. A capacitor acts like a reservoir for electrical charge (Coulombs). Current flows quickly to build up the charge, but it takes time to build up the voltage. *Capacitance* is sometimes considered to be an opposition to the build up of voltage. Capacitance is also considered to be the ability to store charge. The symbol for capacitance is the letter **C**, and the unit is a quantity called the Farad. An electrical device is considered to have a capacitance of one Farad if one Coulomb of charge is stored with one volt between the plates. This Tech Note will discuss the parameters that determine capacitance, different types of capacitors, specifications for capacitors, connecting them in series and in parallel, and how capacitors interact with resistance in a circuit.

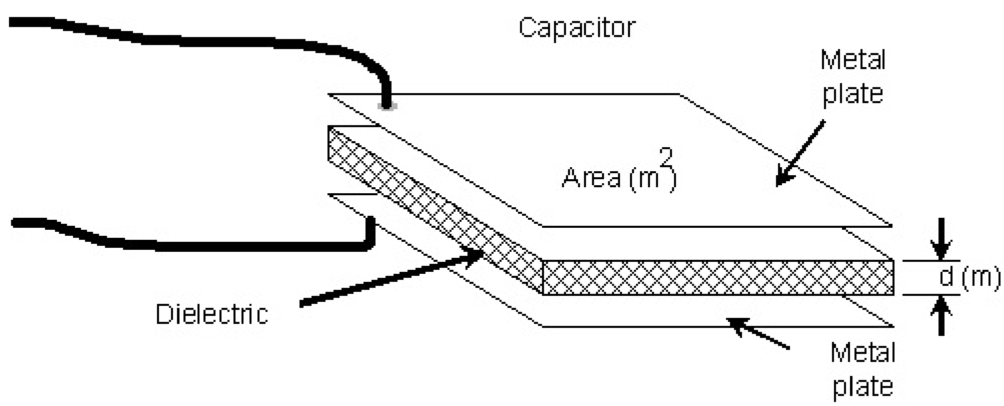

Figure 512.1 *A capacitor consists of two metal plates separated by an insulator called a dielectric. When connected to an electrical source, current flows until the dielectric is fully charged and the voltage between the plates equals the voltage of the source.*

Characteristics of a Capacitor: A *capacitor* consists of two metal plates facing each other with a space between them consisting of an insulating material called a dielectric. The fundamental dielectric material is air or a vacuum which have nearly the same characteristics. Other insulating materials used to build capacitors are rated by a quantity called their *dielectric constant* and it is a value relative to air. For example a certain grade of paper may have a dielectric constant of six. This means it's absolute dielectric constant is six times that of air. The dielectric constant for air or a vacuum is approximately 8.85×10^{-12} Farads/m. A capacitor stores electrical charge. Refer to the diagram of a simplified capacitor in Figure 512.1. When the capacitor is connected to a dc voltage supply, current does not flow through the capacitor. Current flows until the dielectric has reached it's capacity to store charge and the voltage across the capacitor plates is equal to the source voltage. The energy is actually stored in the dielectric as an electrostatic field. The amount of energy that can be stored by a capacitor is directly proportional to the dielectric constant of the insulating material between the plates, and the area of the metal plates, and it is inversely proportional to the distance between the plates (Equation 512.1). The charge (Q) in Coulombs that can be stored by a capacitor is equal to the capacitance (C) in Farads times the voltage between the plates (Equation 512.2). One Farad is equal to one Coulomb per volt. This means that if it takes 10 volts to put one Coulomb of charge in a capacitor dielectric than the capacitance is 0.1 farad (C = 1F/10v). Typical capacitors used in instrumentation have values in the range of microfarads (µf) and picofarads (pf or µµf). Plate area (A) and distance between the plates (d) are in square meters and meters.

$$\textbf{Capacitance (C)} = \frac{\textbf{dielectric constant } (K_r) \times \textbf{plate area (A)} \times (8.85 \times 10^{-12})}{\textbf{distance between plates (d)}} \qquad \textbf{Equation 512.1}$$

$$\textbf{Charge (Q)} = \textbf{Capacitance (C)} \times \textbf{Voltage (E)} \qquad \textbf{Equation 512.2}$$

Increasing the length and width of the capacitor plates will increase the capacitance, however, the result is an increase in the size of the capacitor. Sheets of foil separated by a dielectric material are frequently rolled up into a cylinder to package a large plate area into the smallest size practical. Decreasing the distance between the plates will increase the capacitance while also decreasing the overall bulk of the capacitor. The trade-off of a thin dielectric is that the dielectric strength is also reduced which will limit the peak voltage that can be applied between the plates. The following example will illustrate the effect these dimensions have on the value of a capacitor.

Example 512.1: *Determine the value of a capacitor that has plate dimensions of 20 mm by 300 mm with a distance (d) between the plates of 0.01 mm. The dielectric is a paper material with a dielectric constant (K_r) of 2.0.*

The dimensions are 0.02m by 0.3m by 0.01×10^{-3}m.

$$C = \frac{2 \times (6 \times 10^{-6}\text{m}^2) \times (8.85 \times 10^{-12})}{0.01 \times 10^{-3}\text{m}} = 0.01 \times 10^{-6}\text{ F} = 0.01\mu\text{F}$$

Capacitor Specifications: Capacitors come in a wide variety of types, some with fixed values, some that are adjustable, and some that are made to be variable. Whenever two conductors are close together and separated by an insulating material there will be some capacitance. In many cases this capacitance is insignificant, but in other cases this small capacitance must be taken into consideration. When an actual capacitor is installed in a circuit there are several factors that must be considered. Some capacitors are manufactured in such a way that a particular polarity must be maintained. A type called an electrolytic capacitor usually has a polarity. These types of capacitors will have some marking to indicate the positive and negative terminal. Usually there is a notch on the positive end of the capacitor or there may be a line circling the negative end. In some cases an electrolytic capacitor may have a plus or minus sign marked on the capacitor. The enclosure of an electrolytic capacitor is aluminum and it is always the negative terminal. With a little experience this terminal can be recognized by looking at the ends of the capacitor. Some common markings of an electrolytic capacitor are shown in Figure 512.2. Usually only one of these marking methods is on a capacitor, not all of the markings.

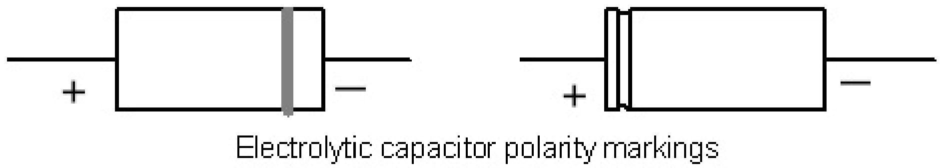

Figure 512.2 *Typical markings that are used to indicate the negative and positive terminal of an electrolytic capacitor.*

For most applications in electronic circuits capacitors are suitable for continuous duty. For some direct current and most alternating current applications where the current level is high, care must be taken to make sure the capacitor will adequately dissipate the internal heat generated. Capacitors for some applications may be marked *intermittent duty* or *continuous duty*. This is especially true when capacitors are used for alternating current applications such as in the starting circuit of a single-phase electric motor. Some capacitors for high current applications are filled with an oil to act as a coolant. Make sure that capacitors used for high current applications have the ability to withstand the operating conditions.

In addition to the value of the capacitors marked usually in microfarads or picofarads, there will usually be a maximum voltage rating. This is known as the *dielectric strength*. This is the maximum peak operating voltage between the plates. If this voltage is exceeded the dielectric will usually break down and one plate will become shorted to the other. When the plates are shorted the capacitor has been ruined. An ohmmeter can usually be used to check for continuity between the plates. The battery in the ohmmeter will cause an initial current flow as the plates are charging to the level of the battery voltage, then current will cease and a high resistance reading indicates the plates are not shorted.

Capacitance is obtained by separating two flat metal plates with an insulating material. It is fairly easy to size the metal plates to a precise tolerance, but it is not so easy to maintain a uniform precise spacing between the plates. To minimize space some capacitors have the two plates and the dielectric and they are rolled up and placed in a container. This packaging technique does not help with maintaining a uniform plate spacing. As a result the *tolerance* of a capacitor is sometimes difficult to control. A capacitor will usually rise in temperature when placed in service and this may also cause some change in the plate area and plate spacing. A capacitor tolerance of ±20% is not uncommon. This means a capacitor marked 10µF could have an actual value as low as 8µF or as high as 12µF.

Capacitor Markings and Codes: Many capacitors are large enough to mark the value on the outside of the capacitor. Typical markings are in micorfarads (µF) or (MFD) or in picofarads (pF) or (µµF). Sometimes the **F** for farads is deleted and only the numeric prefix is shown such as **µ** or **p** following the value. When working with an electronics technician you will need to get used to their jargon such as the work “puff” for picofarads.

There are color codes used to designate the value of a capacitor. These color codes are usually in the form of color dots. If you need to identify a capacitor with a color code you can usually find the information on-line. Another way to identify a capacitor with an unknown value is by using an LC meter. This is a special meter designed to determine the value of a capacitor or an inductor. Some disc capacitors can be very small and a *numeric code* is printed on the capacitor. It is useful to memorize this numeric code as it is frequently encountered and you generally will not have an LC meter available to measure the value. The numeric code has three digits and the value is given in picofarads (pF). The first two digits of the code are the first two digits of the value. The third digit is the number of zeros to place after the first two digits. For example, the code 104 means the value is 100,000 pf or 10^{-5} pF. This is equivalent to 0.1µF. The value 472 has the value 4700 pF or 0.0047µF.

Capacitors Connected in Series and in Parallel: Capacitors can be combined in series and in parallel if necessary to obtain the desired capacitance. Increasing the area of the plates of the capacitor will increase the capacitance according to Equation 512.1. Connecting capacitors in parallel, as shown in Figure 512.3, is like increasing the area of the plates. The total capacitance is the sum of the value of the capacitors connected in parallel as in Equation 512.3.

$$C_T = C_1 + C_2 + C_3$$ **(Capacitors connected in parallel)** **Equation 512.3**

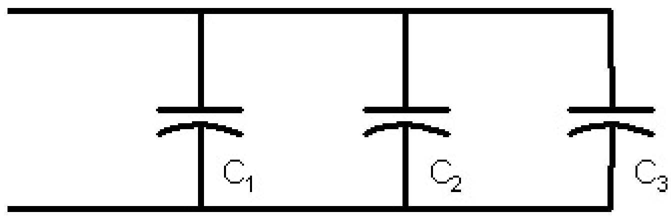

Figure 512.3 *When capacitors are connected in parallel their values add.*

Connecting capacitors in series, as illustrated in Figure 512.4, has the effect of increasing the distance between the plates of the capacitor which will result in the final capacitance being less than the smallest value of capacitor connected in series. The reciprocal of the resulting capacitance of several capacitors in series is the sum of the reciprocals of the values of the individual series connected capacitors according to Equation 512.4. After the reciprocals have been summed, take the reciprocal of the final value to get the resultant capacitance. If the series connected capacitors are all of the same value then all that is required is to divide by the number of capacitors. For example if two 0.68 µF capacitors are connected in series the resultant value will be 0.68 µF divided by 2 which is equal to 0.34 µF.

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$ **(Capacitors connected in series)** **Equation 512.4**

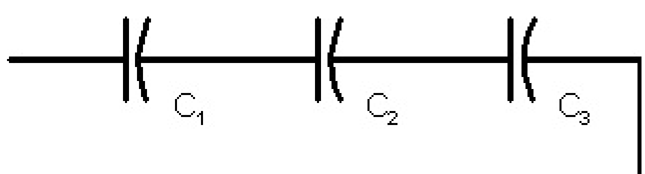

Figure 512.4 *When capacitors are connected in series the reciprocal of their values add.*

Capacitive Reactance: In order to determine the effect of a capacitor on a circuit it is necessary to convert the value of capacitance into a form that is compatible with Ohm's law since a capacitor has a resisting effect on the circuit. The effect of a specific capacitor in a circuit depends upon the frequency of the electrical supply involved. The term used is *reactance* and in this case the quantity is *capacitive reactance*. The unit of capacitive reactance is the Ohm, but capacitive reactance cannot be added arithmetically to resistance. Both are vectors and they act at a right angle to each other. The symbol for reactance is the letter **X** and in particular capacitive reactance has a subscript so it is designated as $\mathbf{X_C}$. Equation 512.5 is used to determine the capacitive reactance given the value of the capacitor in Farads (C) and the frequency of the electrical supply (f) in Hertz.

$$X_C = \frac{1}{2\pi f C}$$ **Equation 512.5**

The current limiting effect a particular capacitor has on a circuit depends upon the frequency of the electrical supply. Note in Equation 512.5 that frequency in Hertz (f) is in the denominator which means that the higher the frequency the smaller will be the capacitive reactance for a given value of capacitance. Here are three examples where a 0.47μF capacitor is placed in a dc circuit (0 Hz), an ac circuit operating at 60 Hz, and an ac circuit operating at 200 kHz. This is summarized in Figure 512.5.

C = 0.47μF = 0.00000047F = 0.47×10^{-6} F

where f = 0 Hz, X_C = infinite ohms

where f = 60 Hz, X_C = 5600 ohm

where f = 200 kHz, X_C = 1.69 ohm

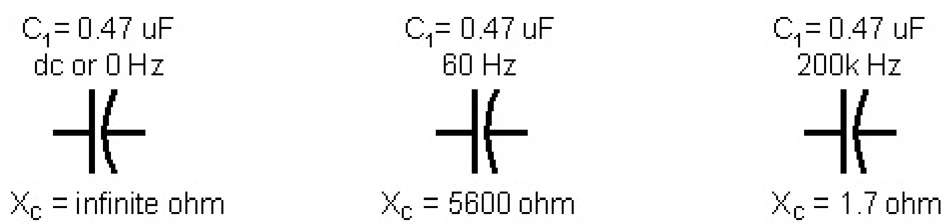

Figure 512.5 *Capacitive reactance of a particular capacitor decreases as the frequency increases.*

In a direct current circuit with a constant voltage the capacitor acts like an open switch preventing current flow. Capacitors are actually used in some circuits to block direct current and remove a dc bias from an alternating signal. At very high frequency the capacitor has very little effect on the circuit. Because the effect of a capacitor is different depending upon the frequency a capacitor can be useful in filtering where multiple frequencies are present in a circuit. The value of capacitance in combination with other components can be chosen to attenuate some frequencies and pass other frequencies. When a capacitor is connected in series with a load resistor (R_L) as shown in Figure 512.6, it passes high frequencies and blocks low frequencies. This is called a high pass filter. When the capacitor is placed in parallel with the load resistor (R_L), also shown in Figure 512.6, it shunts high frequencies past the load resistor so that only low frequencies are forced through the resistor to be detected as a voltage drop across the resistor. This is called a low pass filter.

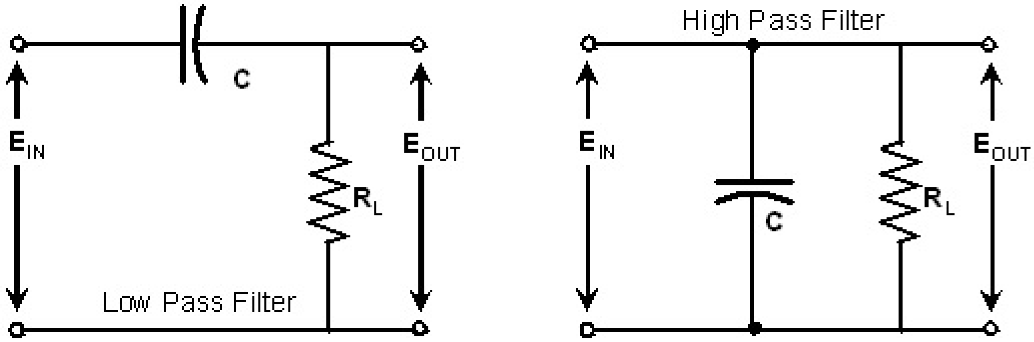

Figure 512.6 *A capacitor can be used in a circuit with multiple frequencies to help select the desired frequencies and reject other frequencies. Connected in series with a load resistor it acts to pass high frequencies and when connected in parallel with the load resistor it acts to reject high frequencies.*

Impedance: In an alternating current circuit the value of capacitive reactance (X_C) is determined using Equation 512.5. Consider an ideal capacitor connected to a dc voltage source where there is no resistance to the circuit other than the internal resistance of the voltage source. This is illustrated in Figure 512.7. The instant the switch is closed current will flow limited only by the resistance of the source. At this instant there will be no voltage across the capacitor plates. As charge is stored by the capacitor a voltage will build up on the plates and the current will decrease. Eventually the voltage will equal that of the source and current will be zero. Note that when the current is at it's maximum the voltage is zero, and when the voltage is at it's maximum the current is zero. This is not the case with a resistor. With a resistor, the voltage and current are zero at the same time and at a maximum at the same time. When the voltage source is a sine wave, with an ideal capacitor the voltage and current sine waves are offset by 90°. With only a resistor in the circuit the voltage and current sine waves are exactly synchronized. The capacitive reactance (X_C) as calculated using Equation 512.5, therefore, acts 90° out of synchronization with any series resistance in the circuit.

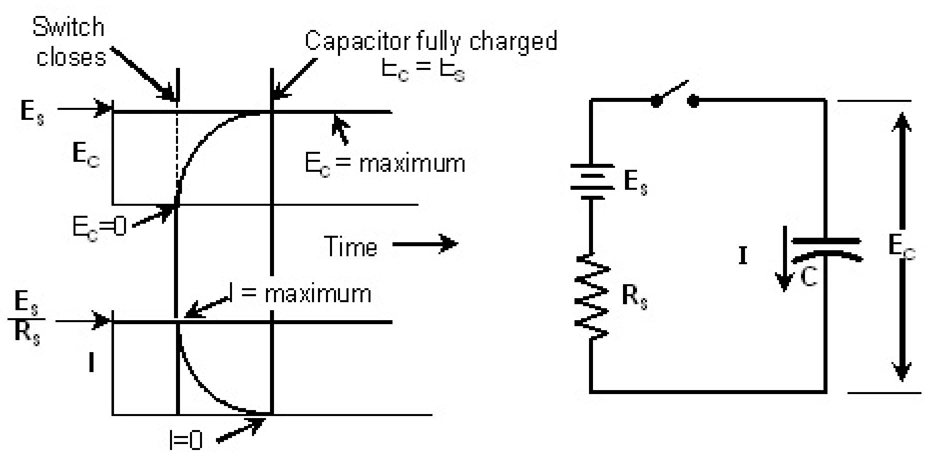

Figure 512.7 *When the switch is closed and a dc voltage is applied to a capacitor, current will flow immediately to start charging the capacitor, and the current will decrease as the charge and voltage build up in the capacitor. Current occurs first and voltage builds up later so that current is leading the voltage in a capacitive circuit.*

When an ac circuit contains both resistance (R) and capacitive reactance (X_C) their combined effect is called *impedance* (Z), but even though they both have units in Ohms their values cannot simply be added to determine their combined effect. When a resistor and a capacitor are connected in series in an ac circuit, an impedance diagram using vectors as resistance (R) and reactance (X_C and X_L) can be used to illustrate how resistance and reactance are combined to determine impedance (Z). Capacitive reactance (X_C) builds up over time as discussed earlier, therefore, in Figure 512.8 capacitive reactance is 90° out-of-phase with resistance and lagging behind the resistance. Capacitive reactance (X_C) points straight down in the impedance diagram with resistance pointing to the right. Inductive reactance (X_L) points straight up and is discussed in Tech Note 513. The resistance and capacitive reactance form the two sides of a right triangle, as illustrated in Figure 512.8, and their net effect or impedance is the hypotenuse of that triangle. From mathematics it is known that the sum of the squares of the two sides of the right triangle is equal to the square of the hypotenuse. Impedance (Z) of an ac circuit can be determined using Equation 512.6 when the values of resistance and capacitive reactance are connected in series. Equation 512.6 only works for the case where the resistance, capacitive reactance, and inductive reactance are connected in series. In a series circuit a capacitor and an inductor cancel the effect of the other and their combined effect is the difference between their reactance values. If the circuit does not contain an inductor, the value of X_L is entered as zero.

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$ **Equation 512.6**

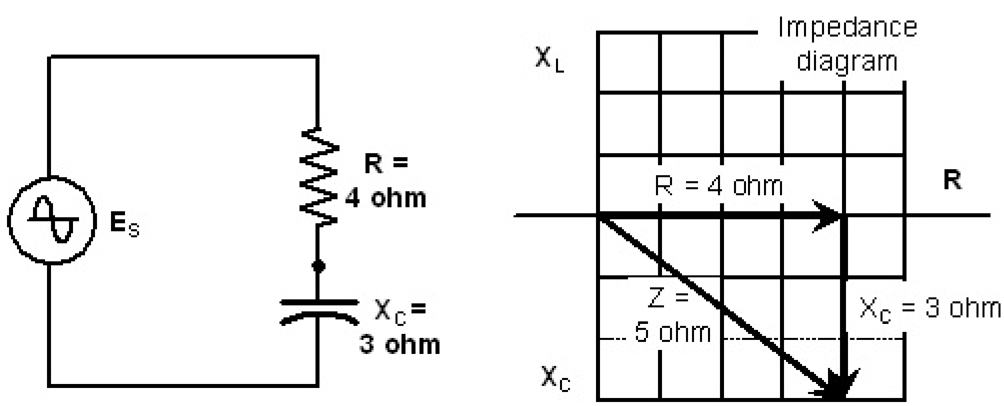

Figure 513.8 *When a resistor and a capacitor are connected in series in a ac circuit the resistance and capacitive reactance act 90° out-of-phase with each other with the capacitive reactance pointing down in an impedance diagram and resistance pointing to the right. The hypotenuse of the right triangle that is formed is the value of their net effect or impedance.*

Time Constant and the RC Circuit: An analogy of how a capacitor works is illustrated in Figure 512.9 where a tank is filled with liquid from a large supply reservoir. When the valve is opened, the rate of flow of liquid into the tank depends upon the liquid head ($\Delta h = h_S - h_T$) and the resistance provided by the connecting pipe. This is illustrated in Figure 512.9. With a capacitor in a circuit, the driving force is the difference between the supply voltage and the voltage on the capacitor plates ($\Delta E = E_S - E_C$) and the resistance of the wire in the circuit. As the tank fills with liquid difference in head, Δh, decreases and the flow rate decreases. The tank fills rapidly at first and then the filling rate decreases. The same is true with a capacitor when subjected to a constant voltage, the difference in voltage is great at first and then decreases as a voltage builds up on the capacitor plates. Likewise, the current flow is very high at first and then decreases and eventually stops as the voltage on the capacitor plates approaches the voltage of the supply. When the supply is a constant voltage (E_S), the voltage on the capacitor plates (E_C) builds up in an exponential manner according Equation 512.7, where *e* is 2.718, *R* is the resistance of the circuit in Ohms, *C* is the value of capacitance in Farads, and time (*t*) is in seconds. The voltage across the capacitor will build up as illustrated in Figure 512.7 and Figure 512.10.

$$E_C = E_S\left(1 - \frac{1}{e^{\frac{t}{RC}}}\right)$$ **Equation 512.7**

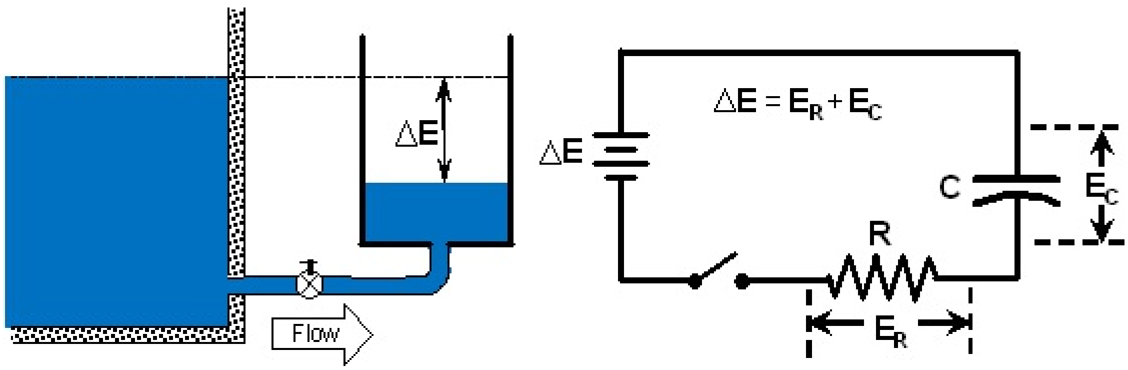

Figure 512.9 *The charging of a capacitor is like filling a tank with liquid from a large reservoir where the pressure or head decreases as the tank fills thus resulting in a progressively decreasing liquid flow. Likewise as a voltage develops between the capacitor plates the difference between the capacitor voltage and supply voltage decreases resulting is a decrease in current flow and the voltage build-up on the capacitor follows the exponential relationship of Equation 512.7.*

Note in Equation 512.7 that the exponent for the value *e* is *t/RC*. The product of resistance and capacitance is time in seconds (C in Farads is Coulombs^2/Joule, and R in Ohms is Joule seconds/Coulomb^2). It is interesting to note that when *t = RC* the exponent is 1 and the value is simply *e* (2.718). The reciprocal of e^1 is 0.368 which subtracted from 1 gives 0.632. This means that in a time interval of *t = RC*, the voltage across the capacitor (E_C) will reach 63.2% of the supply voltage (E_S). It is further observed that when *t = 5RC* the reciprocal of e^5 is 0.006 and subtracted from 1 gives a value of 0.994. This means that for all practical purposes, the voltage across the capacitor (E_C) is equal to the supply voltage (E_S) when the time elapsed is equal to 5 times the product of the resistance (*R*) and the capacitance (*C*). The product of the resistance and capacitance in a series circuit is called the *time constant* (τ). In one time constant the voltage across the capacitor has reached 63.2% of the supply voltage and the current has decreased to only 36.8% of it's original maximum value. In five time constants the voltage across the capacitor has essentially matched the supply voltage and the current has decreased to zero. These values are illustrated in Figure 512.10 with voltage percentages also shown for time constants of $\tau = 2$ (86.4%), $\tau = 3$ (95.0%), and $\tau = 4$ (98.2%). Many natural phenomenon behave like a series resistor-capacitor circuit and exhibit this same exponential relationship. For these situations the time constant can be determined experimentally. A series resistor-capacitor circuit (RC circuit) is frequently used in electronics for timing when exact precision is not necessary. The charging rate of the RC circuit can be quite accurate, however, adjustments may be necessary due to the precision of resistor and capacitor values.

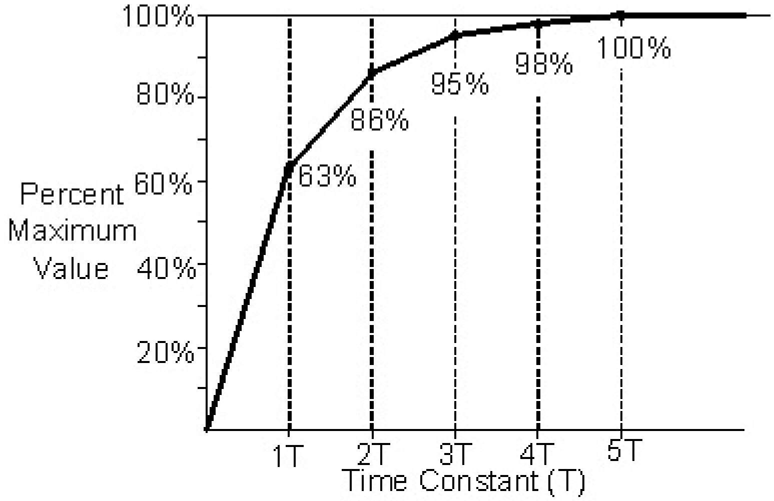

Figure 512.10 *When exposed to a constant dc voltage, the build up of voltage across the plates of a capacitor in a resistor-capacitor series circuit follows a predictable exponential curve and reaches essentially maximum voltage in five time constants where the time constant is equal to the product of the resistance and capacitance.*

When the supply voltage is a constant value and the voltage across the plates of the capacitor matches the supply voltage there is no current flow in the circuit. According to Equation 512.2 the charge in Coulombs stored by a capacitor is equal to the product of the capacitance in Farads and the voltage across the plates. Current in Amperes is the time rate of flow of charge in Coulombs per second. Current flow in a capacitor circuit implies that the level of charge is changing, and since the capacitance is a fixed value, according to Equation 512.2 the voltage must be changing with time. This concept gives rise to Equation 512.8 which shows that current flow in a capacitor circuit is equal to the product of capacitance and the time rate of change of voltage. Refer to Figure 512.7 and note that when the voltage across the capacitor plates is changing very fast the current flow is at it's maximum, and when the capacitor voltage reaches a maximum and is no longer changing with respect to time the current is zero.

$$i = C\frac{dv}{dt}$$ **Equation 512.8**

Think back to the liquid analogy of a capacitor in Figure 512.9 where flow over a period of time was necessary to fill the tank. Likewise, current flow over a period of time is necessary to deliver charge to the capacitor where the ratio of capacitor voltage to supply voltage is a measure of the state of charge of the capacitor. Solving Equation 512.8 for voltage involves rearranging the equation and integrating the current over time as shown in Equation 512.9. Since current is the time rate of delivering charge to the capacitor it is necessary to integrate current over a time interval to determine the quantity of charge delivered to the capacitor. The value of capacitance is the quantity of charge that must be delivered to the capacitor to develop one volt between the plates. Dividing the quantity of charge delivered by the capacitance gives the voltage that developed across the capacitor plates during any time interval. This can be verified with a simple dimensional analysis where current (i) is in Coulombs per second, capacitance (C, Farads) is in Coulombs^2 per Joule, and Δt is in seconds.

$$v = \frac{1}{C}\int i dt$$ **Equation 512.9**

Energy Stored in a Capacitor: Capacitance (C in Farads) is the quantity of charge stored in the dielectric for each volt developed between the plates. If a capacitor is rated 2200 μF then 2.2×10^{-2} Coulombs of charge is stored in the dielectric for each volt across the plates. If there is 100 volts between the capacitor plates the charge stored is 0.2 Coulombs. This is potential energy in the form of electrical charge. Short the two terminals of the capacitor and current will flow. The product of current and voltage is power. Substitute Equation 512.9 for current flow in a capacitive circuit into the relationship for power and the result is Equation 512.10 where power in a capacitive circuit is the product of capacitance, the voltage, and the time rate of change of voltage. A simple dimensional analysis performed on Equation 512.10 results in Joules per second which is watts (C × v × dv/dt = Coulombs^2/Joule × Joules/Coulomb × Joules/Coulomb second).

$$p = vi = Cv\frac{dv}{dt}$$ **Equation 512.10**

Energy or work is power expended over a period of time. The amount of energy that can be stored in a capacitor can be determined by integrating Equation 512.10 over time. In the case where the capacitor is connected to a constant voltage source the result is Equation 512.11. The energy in Joules stored by a capacitor is equal to one-half the product of the capacitance and the square of the voltage. The following example will demonstrate how to determine the energy stored in the dielectric of a capacitor.

$$W = \int p dt = \int Cv\frac{dv}{dt}dt = \int Cv dv = \frac{1}{2}Cv^2$$ **Equation 512.11**

Example 512.2: *Determine the energy stored in a power capacitor with a rating of 1000μF with 100 volts across the capacitor plates.*

Use Equation 512.11 where $C = 10^{-3}$F and v is 10^2 volts.

$W = 0.5 \times 10^{-3}\text{F} \times (10^2\text{v})^2 = 5\text{Joules}$

CAUTION: Capacitors used in power applications and in some portions of electronic circuits can store enough charge at high enough voltage to be dangerous. Capacitors can store charge for long periods of time. Never assume a capacitor is discharged. For high power applications capacitors must be discharged through properly selected power resistors.