Electrical Tech Note — 501

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Logarithms and Exponents

Working with electronics components and circuits requires a basic understanding of arithmetic operations (adding, subtracting, multiplying, dividing) using exponential numbers ($10^X = n$) and logarithms ($\log_{10} n = X$). For example it may be necessary to multiply 470,000 and 0.000068. The value 470,000 can be represented in various ways such as 47×10^4 or 4.7×10^5 or 0.47×10^6. Let's represent 0.000068 as 68×10^{-6} and 470,000 as 0.47×10^6. The exponent indicates how many digits to the left or right to move the decimal point. Multiplying the two previous numbers results in a value of 31.96.

$$(0.47 \times 10^6) \times (68 \times 10^{-6}) = 0.47 \times 68 \times (10^{+6+(-6)}) = 31.96 \times 10^0 = 31.96$$

Powers of 10: An exponent tells how many times the value is to be multiplied by itself. In this case the value is 10. Any number raised to the power 0 is the value 1. It doesn't matter how big or how small the number, if it is raised to the 0 power the value is 1.

This discussion is limited to only powers of 10. When the power is positive (+) the exponent gives the number of zeros to add to the right of the value 1. Table 1 gives the positive powers of 10 along with their common prefix and abbreviation.

Table 1. Positive powers of ten and the common prefix and abbreviation.

Power of 10	The number	Prefix	Abbreviation	
10^0	1			
10^1	10	deca	da	
10^2	100	hecto	h	hundred
10^3	1,000	kilo	k	thousand
10^4	10,000			
10^5	100,000			
10^6	1,000,000	mega	M	million
10^7	10,000,000			
10^8	100,000,000			
10^9	1,000,000,000	giga	G	billion

When the exponent is a negative whole number the exponent is the number of spaces to move the decimal to the left of the value 1. Actually it is the exponent minus 1 zeros to the left of the value 1. This is shown in Table 2.

Table 2. Negative powers of ten and the common prefix and abbreviation.

Power of 10	The number	Prefix	Abbreviation
10^{0}	1.0		
10^{-1}	0.1	deci	d
10^{-2}	0.01	centi	c
10^{-3}	0.001	milli	m
10^{-4}	0.000,1		
10^{-5}	0.000,01		
10^{-6}	0.000,001	micro	µ
10^{-7}	0.000,000,1		
10^{-8}	0.000,000,01		
10^{-9}	0.000,000,001	nano	n
10^{-10}	0.000,000,000,1		
10^{-11}	0.000,000,000,01		
10^{-12}	0,000,000,000,001	pico	p or µµ

Adding and Subtracting Numbers Represented as Powers of 10: In order to add or subtract two numbers that are represented as powers the two or more numbers must be put in a form that they have the same power of ten. For example add 3.3×10^{4} to the number 2.2×10^{3}. Before the two numbers can be added the powers must be the same. Let's change 3.3×10^{4} to 33×10^{3}. Now the two numbers can be added to get 35.2×10^{3} which is 35,200. Here is another example where we want to add 0.0047×10^{-6} to the number 6800×10^{-12}. In this case let's convert 6800×10^{-12} to 0.0068×10^{-6}. The sum of the two values is 0.0115×10^{-6}.

Multiplying Numbers Represented as Powers of 10: When two numbers are represented as powers of 10, simply multiply the values and add the exponents. (Equation 501.1) Take the values 3.3×10^{4} and multiply it by 2.2×10^{3}. The product of these two numbers is $3.3 \times 2.2 \times 10^{4+3}$ which is equal to 7.26×10^{7}. When one exponent is positive and the other is negative, simply subtract the two exponents such as 6.8×10^{-6} added to the number 2.2×10^{5}. The result is 14.96×10^{-1} which is more conveniently stated as 1.496.

$$(A \times 10^{x}) \times (B \times 10^{y}) = (A \times B) \times 10^{x+y} \qquad \textbf{Equation 501.1}$$

Where A and B are 1 the equation simplifies to $10^{x} \times 10^{y} = 10^{x+y}$

It is fairy easy to estimate the value of a power of 10 that is not an integer power by using the principle of Equation 501.1 and breaking the number up into two parts. To do this it is necessary to recognize that $10^{0.3}$ is approximately 2 (1.99), $10^{0.6}$ is approximately 4 (3.98), and $10^{0.9}$ is approximately 8 (7.94). This is also summarized in Table 3. Break the problem into two parts consisting of an integer power of 10 and a remainder that is between zero and one. As an example determine the approximate value of $10^{1.3}$. Using Equation 501.1, this number can be separated into two parts $10^{1+0.3} = 10^{1} \times 10^{0.3}$. The value of 10^{1} is 10 and the value of $10^{0.3}$ is 2. Now multiply these numbers together to get the approximate answer which is 20.

$$10^{1.3} = 10^{1+0.3} = 10^{1} \times 10^{0.3} \approx 10 \times 2 \approx 20$$

Try one more example of a power of 10 that is not an integer. This time determine the approximate value of $10^{2.6}$. This can be separated into $10^2 \times 10^{0.6}$. It is necessary to remember that the value of $10^{0.6}$ is double the value of $10^{0.3}$ which is 2 × 2 or 4. The approximate answer is 100 times 4 or 400.

$$10^{2.6} = 10^{2+0.6} = 10^2 \times 10^{0.6} \approx 100 \times 4 \approx 400$$

Dividing Numbers Represented as Powers of 10: When one number represented as a power of 10 is divided by another number represented as a power of 10, divide the numbers and subtract the exponent in the denominator from the exponent in the numerator (Equation 501.2). Another way to say this is to change the sign of the exponent in the denominator and add it to the exponent in the numerator. As an example, divide 5 by 2×10^3. The value 5 can be represented by 5×10^0. The answer is 2.5×10^{-3} which is 0.0025.

$$\frac{5}{2 \times 10^3} = \frac{5 \times 10^0}{2 \times 10^3} = 2.5 \times 10^{0-3} = 2.5 \times 10^{-3} = 0.0025.$$

$$\frac{A \times 10^x}{B \times 10^y} = \frac{A}{B} \times 10^{x-y} \qquad \textbf{Equation 501.2}$$

What is a Logarithm? There are common logarithms and natural logarithms and this discussion deals only with common logarithms. The common logarithm of a number is the power to which 10 is raised to get the number. For example the logarithm of 100 is 2 because 10^2 is 100. When the number is the digit 1 followed by zeros, the logarithm is easy because it is the number of zeros. For example, the logarithm of 10,000 is 4, the logarithm of 10 is 1, and the logarithm of 1 is zero. When the number is less than 1 the logarithms are negative numbers. For example, the logarithm of 0.1 is −1 and the logarithm of 0.01 is −2 etc. A logarithm scale is graphed in Figure 1. Note that each time the number doubles, the difference in the logarithm of that number 0.3.

$10^X = n$ $\qquad$ $10^2 = 100$

$\log_{10} n = X$ $\qquad$ $\log_{10} 100 = 2$

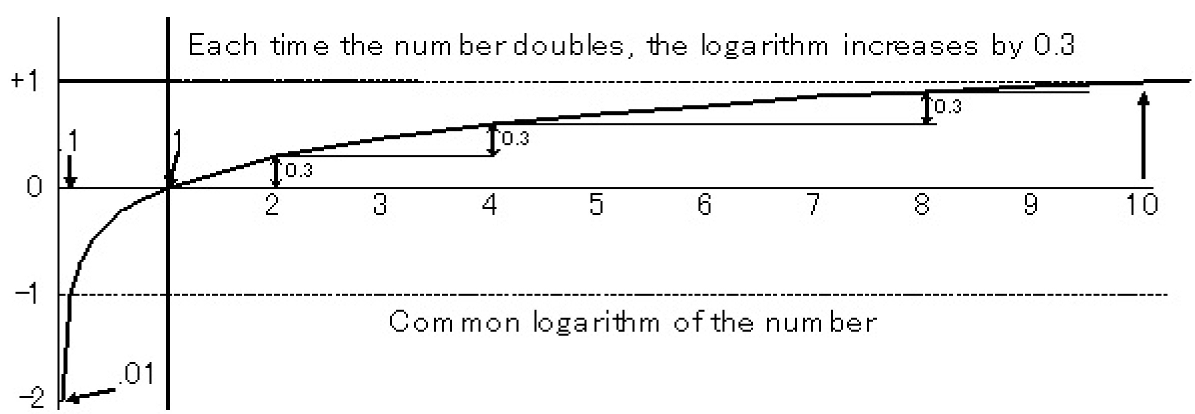

Figure 1. *Graph of the common logarithm of numbers. If the graph is extended, the logarithm of 100 is +2, and the logarithm of 1000 is +3.*

If the number is not an integer power of 10 then the logarithm will be between the integer power of 10 below and the integer power of 10 above. Let's use 80 as an example. The logarithm of 80 will be greater than 1 (log 10) and less than 2 (log 100). The value can be looked up in a table or determined with a calculator, but for electronics work it is important to understand the concept of logarithms. We will return to this problem later, but first it is necessary to review a few facts about logarithms.

Common logarithms of numbers are summarized in Table 3. The bottom portion of the table gives the logarithms for numbers between 1 and 10. These values are approximate. The actual logarithms are shown in parenthesis. Note that each time the number doubles, the logarithm of that number increases by 0.3. The value 2 is twice 1 and the difference in logarithm is 0.3. The value 4 is twice 2 and the difference in logarithm is 0.3. The value 8 is twice 4 and the difference in logarithm is 0.3.

Table 3 How to Determine the Approximate Logarithm of a Number.

x					Log x (n)	10^n		
1	Log 1	=	Log 10^0	=	0	10^0	=	1
10	Log 10	=	Log 10^1	=	1	10^1	=	10
100	Log 100	=	Log 10^2	=	2	10^2	=	100
1,000	Log 1,000	=	Log 10^3	=	3	10^3	=	1,000
10,000	Log 10,000	=	Log 10^4	=	4	10^4	=	10,000
x	Log x	=	Log 10^n	=	n	10^n	=	1×10^n
1	Log 1	=	Log 10^0	=	0	10^0	=	1
2	Log 2			=	0.3 (0.301)	$10^{0.3}$	=	2 (1.99)
4	Log 4			=	0.6 (0.602)	$10^{0.6}$	=	4 (3.98)
8	Log 8			=	0.9 (0.903)	$10^{0.9}$	=	8 (7.94)
10	Log 10	=	Log 10^1	=	1	10^1	=	10

In order for the human ear to perceive a difference in sound level, the sound level must change approximately 26%. Human ear response to sound level is exponential, and the logarithm of the value 0.26 is approximately 0.1. This means that if the logarithm of the sound level changes by a value of 0.1 the average human can tell the sound level changed. The bel (B) is the logarithm of the actual sound level, and ten decibels make up one bel (0.1 B = 1dB). The average human can perceive a sound level change of 1 dB. If a sound level changes from 80dB to 81dB a human can perceive the increase. If the sound level changes from 80dB to 83dB the actual sound level doubled.

When the logarithm is to be determined for a value that is not an integer power of 10, the logarithm of the number can be estimated using the values from the lower portion of Table 3 and the mathematical rules for logarithms. First let's review the math rules for logarithms. The log of the product of two numbers is the log of one number plus the log of the other number (Equation 501.3). The log of one number divided by another number is the log of the numerator minus the log of the denominator (Equation 501.4). The log of a number raised to a power is the log of the number multiplied by the power (Equation 501.5).

$$\text{Log } xy = \text{Log } x + \text{Log } y \qquad \textbf{Equation 501.3}$$

$$\text{Log } \frac{x}{y} = \text{Log } x - \text{Log } y \qquad \textbf{Equation 501.4}$$

$$\text{Log } y^n = n \times \text{Log } y \qquad \textbf{Equation 501.5}$$

Now back to the example where we wish to determine the logarithm of the number 80. The following method of determining the logarithm of a number is only approximate, but it helps to understand the concept which is important in electronics work. Divide the number by the largest integer power of 10 that is smaller than the number. For this example that value is 10. The number 80 divided by 10 is 8. Using Equation 501.3, Log 80 is equal to Log 10 plus Log 8. Refer to the values in Table 3. The log of 10 is 1 and the log of 8 is 0.9. Adding these two numbers gives Log 80 = 1.9. It is recommended that a person working with electronics memorize the values of Log 2, Log 4, and Log 8. This means that $10^{1.9}$ is equal to 80.

$$\frac{80}{10} = 8$$

$$\text{Log } 80 = \text{Log } (10 \times 8) = \text{Log } 10 + \text{Log } 8 = 1 + 0.9 = 1.9$$

Try one more example. Determine the value of Log 4000. First determine that the largest integer power of 10 that is smaller than 4000 is 1000. The log of 1000 is 3, therefore the final answer will be some fraction larger than 3 but less than 4. Divide 4000 by 1000 to get 4. The task is to determine Log (1000 × 4). Using Equation 501.3, Log (1000 × 4) is equal to Log 1000 plus Log 4. Log 2 is 0.3 and Log 4 is just double that value or 0.6. The answer is Log 4000 is equal to Log 1000 plus Log 4 which is equal to 3 plus 0.6 or 3.6. This means that $10^{3.6}$ is equal to 4000.

$$\frac{4000}{1000} = 4$$

$$\text{Log } 4000 = \text{Log } (1000 \times 4) = \text{Log } 1000 + \text{Log } 4 = 3 + 0.6 = 3.6$$

Conclusions: Values of components used in electronics can be very large and very small thus requiring the use of numbers with exponents. It is necessary to be able to add, multiply, and divide numbers with exponents.

Power gain and voltage gain of amplifiers is frequently given in decibels (dB). By taking the ratio of the output power or voltage of an amplifier and dividing it by the input power or voltage, the gain is determined. Frequently the gain is expressed in decibels (dB). A decibel (dB) is one tenth of a bel (B). Or it takes ten decibels to equal one bel (10 dB = 1B). A bel (B) is the logarithm of the power or voltage gain. Multiply bels by 10 to convert to decibels (dB). Power gain and voltage gain of amplifiers is discussed in Tech Note 502. Even though voltage gain and power gain are logarithmic they are not equivalent. This will be explained in Tech Note 502.

Practice Problems: (Do not use a calculator to determine exponents or logarithms)

1. Time constant in seconds for a series RC circuit is the product of R and C. If R is 100,000 ohm and C is 0.1 µf, determine the time constant in seconds.

2. Two resistors with values of 2.2×10^6 ohm and 330,000 ohm are connected in series their total resistance is the sum of the two values. Determine the sum of the two resistor values.

3. Two capacitors are connected in parallel and the total capacitance is the sum of the two values. The capacitor values are 0.0068 µf and 10,000 µµf. Determine the total capacitance of the circuit.

4. The time constant for an LR series circuit is the inductance in henries divided by the resistance in ohms. The inductance is 10 millihenries and the resistance is 2000 ohm. Determine the time constant of the circuit in seconds.

5. Determine the value of 10 raided to the 3.3 power ($10^{3.3}$).

6. Determine the logarithm of the value 800.

7. The power gain of an amplifier is 40 dB. Determine the power gain of the amplifier in bels.

8. One amplifier has a power gain of 40 dB and another has a power gain of 46 dB. How much more power gain is obtained from the second amplifier than the first amplifier?

Answers to Practice Problems:

1. $(1 \times 10^5) \times (0.1 \times 10^{-6}) = 0.1 \times 10^{-1} = 0.01$ seconds

2. $(2.2 \times 10^6) + (0.33 \times 10^6) = 2.53 \times 10^6$ Ohm

3. $(0.0068 \times 10^{-6}) + (0.01 \times 10^{-6}) = 0.0168 \times 10^{-6}$ f

4. $$\frac{10 \times 10^{-3}}{2 \times 10^3} = 5 \times 10^{-6} \text{ seconds or } 5\mu s$$

5. $10^3 \times 10^{0.3} = 1000 \times 2 = 2000$

6. $$\frac{800}{100} = 8$$

   Log 100 + Log 8 = 2 + 0.9 = 2.9

7. $$\frac{40 \text{ dB}}{10} = 4 \text{ B}$$

8. Either convert dB to B and notice that the change is 0.6 which is 2 × 2 = 4,
   or recognize that 3 dB is equal to 0.3B which gives 2 × 2 = 4.