Electrical Tech Note — 222B

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RLC Series and Parallel Circuit Analysis

An RLC circuit consists of components that are resistors (R), inductors (L), and capacitors (C) in an alternating current circuit where the voltage source is a sine wave. For the purpose of this discussion, the voltage source will be considered to be 60 Hz and the voltage and current levels will be the rms values. Often circuits made up of these components can be analyzed fairly easily. It may be useful to review *Tech Note 221* which discusses capacitors and inductors in alternating current circuits along with a brief discussion of polar and rectangular representation of voltage, current, and impedance.

Series Connection of a Resistor, Capacitor, and Inductor: A resistor, capacitor, and inductor are shown connected in series for the circuit of *Figure 222.1.* There is only one path through this series circuit, and there will be only one current. The same current passes through all of the circuit elements. In order to determine the current flow in the circuit it is necessary to determine the impedance (Z) of the circuit. Then Ohm's law can be used to determine the current.

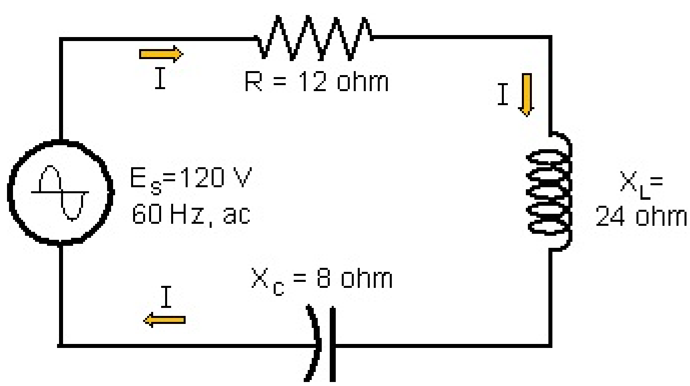

Figure 222.1 *A resistor with a value of 12 ohms is connected in series with a capacitor with a capacitive reactance of 8 ohms and an inductor with an inductive reactance of 24 ohms.* For this series circuit the same current flows everywhere in the circuit.

The resistance (R), capacitive reactance (X_C), and inductive reactance (X_L) of the circuit of *Figure 222.1* are shown plotted on the impedance diagram of *Figure 222.2.* Inductive reactance(X_L) is plotted straight up, capacitive reactance(X_C) is plotted straight down, and resistance(R) is plotted to the right. For a series circuit, resistance, capacitive reactance, and inductive reactance can be added as vectors, and when finished the plot will be a "right triangle." The vertical side of the right triangle will be the difference of the inductive reactance and the capacitive reactance (X_L - X_C). The horizontal side will be the resistance (R). Impedance (Z) will be the hypotenuse of the right triangle and is equal to the square root of the sum of the squares of the two sides of the triangle as represented by *Equation 222.1.* The value of the impedance for the circuit of *Figure 222.1* is worked out on the next page and has a value of 20 ohm.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \textbf{\textit{Eq. 222.1}}$$

$$Z = \sqrt{12^2 + (24-8)^2} = \sqrt{144+256} = \sqrt{400} = 20ohm$$

The impedance Z is the hypotenuse of the right triangle of *Figure 222.2* and forms an angle with the horizontal axis of the triangle. This angle can be determined by finding the inverse tangent (tan-1) of the ratio of the vertical side (X) and the horizontal side of the right triangle (R) as shown in *Equation 222.2.* When making a power calculation for this circuit the power factor (pf) is the cosine of this angle. The calculation to determine the magnitude of te impedance (Z) of this circuit and the angle are shown below.

$$\theta = \tan^{-1}\frac{X}{R} \qquad \textbf{\textit{Eq. 222.2}}$$

magnitude of $Z = \sqrt{12^2 + 16^2} = 20ohm$

angle of Z is $\theta = \tan^{-1}\frac{16}{12} = 53^{\circ}$

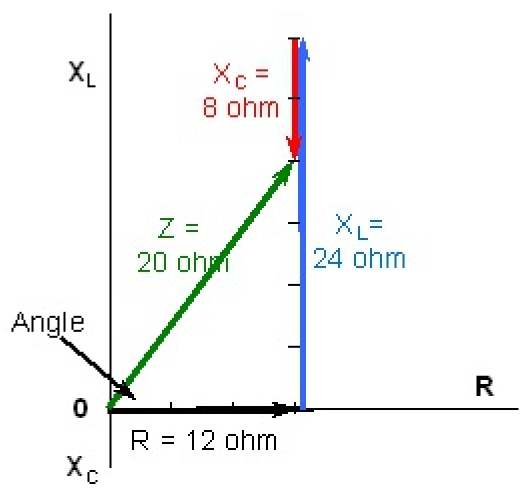

Figure 222.2 *For a series circuit the resistance, capacitive reactance, and inductive reactance can be added to form the legs of a right triangle. The impedance will be the hypotenuse of the triangle.*

Now that the supply voltage and circuit impedance are known, the circuit current can be determined using Ohm's law as indicated by *Equation 222.3*, however, impedance is used rather than resistance. The value of the current for the circuit of *Figure 222.1* is 6 amperes. For a brief review of math in polar form refer to *Tech Note 221*.

$$I = \frac{E}{Z} \qquad Current = \frac{120V}{20ohm} = 6A \qquad \textbf{\textit{Eq. 222.3}}$$

Parallel Connection of a Resistor, Inductor, and Capacitor: When the resistor, capacitor, and inductor are connected in parallel the resistance (R), capacitive reactance (X_C), and inductive reactance (X_L) cannot be added to obtain the impedance of the circuit. The reciprocal of the circuit impedance is the sum of the reciprocals of the impedances of each parallel branch of the circuit, but this process involves more complex mathematics. When the parallel circuit is simple as shown in *Figure 222.3*, consisting of resistance in one branch, inductive reactance in another branch, and capacitive reactance in the still another branch, the total circuit current is fairly easy to determine. Use Ohm's law to determine the current in each branch of the parallel circuits (*Eq.4, Eq. 5, Eq. 6*). Next determine the total current of the circuit using *Equation 222.7.* Finally divide the circuit voltage by the total current to obtain the impedance (Z) of the parallel circuit as indicated by *Equation 222.8.*

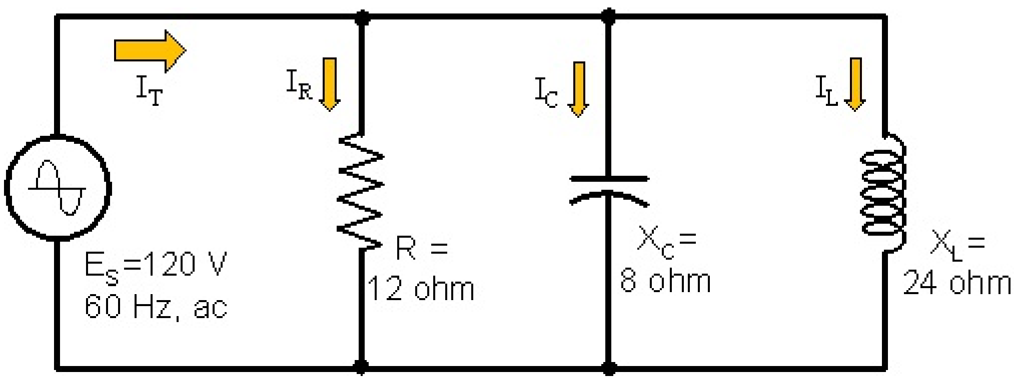

Figure 222.3 *A resistor with a value of 12 ohms is connected in parallel with a capacitor with a capacitive reactance of 8 ohms and an inductor with an inductive reactance of 24 ohms.*

$$I_R = \frac{E_S}{R} \qquad I_R = \frac{120V}{12ohm} = 10A$$ ***Eq. 222.4***

$$I_C = \frac{E_S}{X_C} \qquad I_C = \frac{120V}{8ohm} = 15A$$ ***Eq. 222.5***

$$I_L = \frac{E_S}{X_L} \qquad I_L = \frac{120V}{24ohm} = 5A$$ ***Eq. 222.6***

The current through each branch of this parallel circuit (*Figure 222.3*) must be added as vectors. The inductive branch current (IL) and the capacitive branch current will subtract from each other to form the vertical side of a right triangle. The current through the resistor will be the horizontal side of the right triangle. The total current will be the hypotenuse of the right triangle determined using *Equation 222.7.*

$$I_T = \sqrt{I_R^2 + (I_C - I_L)^2}$$ ***Eq. 222.7***

$$I_T = \sqrt{10^2 + (15 - 5)^2} = \sqrt{100 + 100} = \sqrt{200} = 14.1A$$

The impedance of the circuit of *Figure 222.4* with the resistor, capacitor, and inductor connected in parallel is determined by dividing the supply voltage by the total current using *Equation 222.8.* For the parallel circuit example of *Figure 222.4* the impedance of the circuit is 8.5 ohm with an angle of minus 45º.

$$Z = \frac{E_S}{I_T} \qquad Z = \frac{120V}{14.1A} = 8.5ohm$$ ***Eq. 222.8***

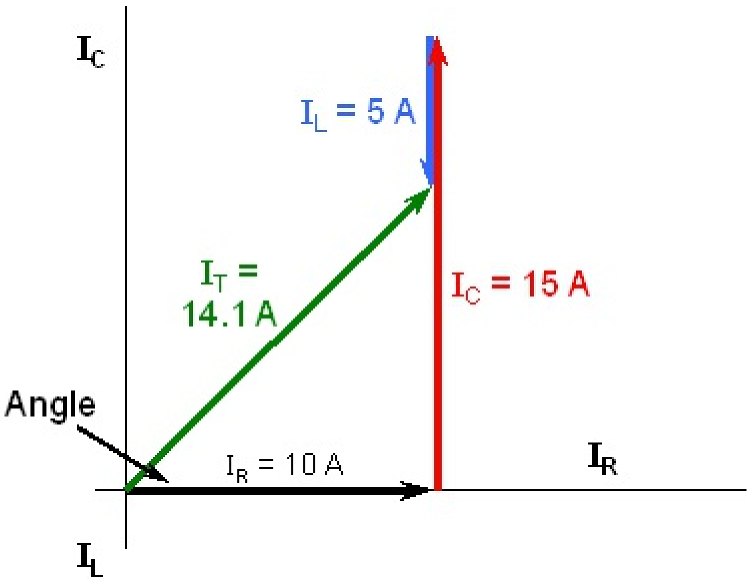

Figure 222.4 *When connected in parallel, the currents through the resistor, capacitor, and inductor can be added with the net current through the capacitor and inductor forming the vertical side of a right triangle on the current diagram. The total circuit current will be the hypotenuse of the triangle.*

Current in an RLC Circuit: The process of building an inductor involves a length of wire usually wound around a laminated steel core. The length of wire will have a significant resistance which means it is generally not possible to separate the resistance from the inductive reactance. *Figure 222.5* has one branch with a resistor in series with an inductor that represents an inductor/resistor circuit similar to an electric motor circuit. The impedance (Z) of the RL circuit is 13 ohms, and when the circuit voltage is divided by the circuit impedance the circuit current is 18.5 amperes. The power factor (pf) of this circuit is 0.39. The calculations are shown later.

To offset the inductance of the circuit sometimes a capacitor is connected in parallel with the RL branch of the circuit. The purpose of this process is to increase the power factor (pf) to a higher value closer to 1.0.

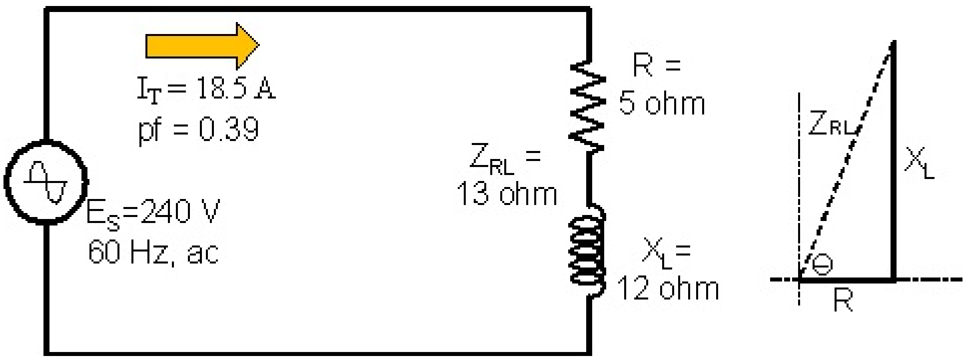

Figure 222.5 *This is an inductive circuit with a resistance (5 ohm) in series with an inductive reactance (12 ohm). The circuit is supplied 240 volts.*

First determine the impedance (Z) of this RL circuit so the total current of the circuit can be determined.

$$Z_{RL} = \sqrt{R^2 + X_L^{\ 2}} = \sqrt{5^2 + 12^2} = \sqrt{169} = 13ohm \qquad \text{(Eq. 221.5)}$$

Next use Ohm's law to determine the current flow of the circuit. Divide 120 V by 13 ohm.

$$I_{RL} = \frac{240V}{13ohm} = 18.5A \qquad \text{(Eq. 222.3)}$$

In order to make a diagram of the current for this circuit it is necessary to determine the angle (θ) of the RL circuit current with respect to the horizontal axis. Because the current is inductive (coil of wire) the angle will be negative. The current vectors are plotted in *Figure 222.6.*

$$\theta = Tan^{-1}\frac{X_L}{R} = Tan^{-1}\frac{12}{5} = Tan^{-1}2.4 = 67°$$

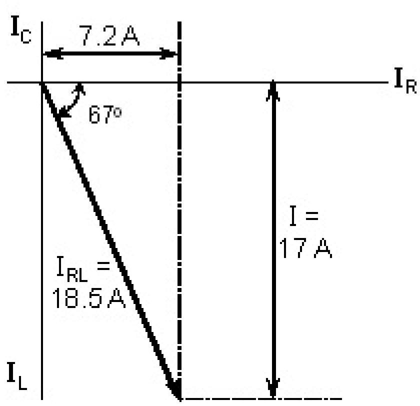

Figure 222.6 *The current for the circuit of Figure 222.6 is plotted downward at an angle of 67°. This makes a right triangle with the hypotenuse of 18.5 A. The vertical and horizontal sides of this triangle can be determined using trigonometry.*

Figure 222.7 shows a capacitor connected in parallel with the RL branch of the previous circuit. The numbers are on the diagram and the calculations are shown later. The capacitor draws 12 amperes and the RL circuit draws 18.5 amperes, but believe it or not the total circuit current is only 8.8 amperes and the power factor (pf) has been raised from 0.39 to 0.82. The values of the current in this circuit with the capacitor added are shown in *Figure 222.8* and the calculations are shown below. This process is called *power factor correction* and the purpose is to reduce the total current flow required to power a load. Manufacturers design power factor correction into individual equipment and electricians can install capacitors on a complete building or portion of a building to increase the power factor. This decreases the current flow necessary to deliver the electrical power.

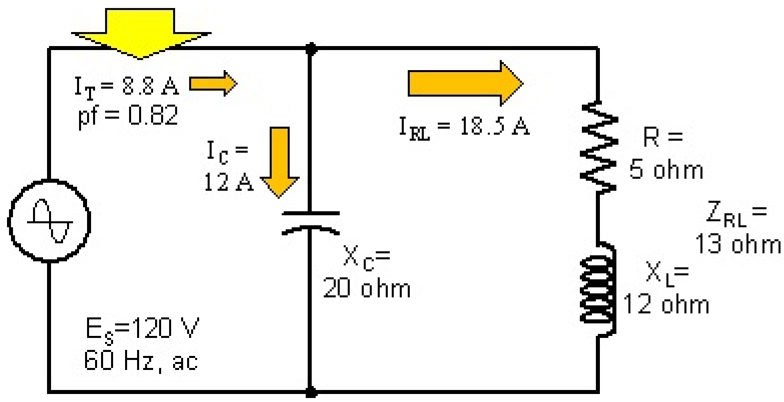

Figure 222.7 *A resistor with a value of 9 ohms is connected in series with an inductor that has an inductive reactance of 12 ohms. Connected in parallel with the RL branch is a capacitor with a capacitive reactance of 20 ohms.*

Figure 222.8 is a continuation of *Figure 222.6.* The current flowing through the capacitor is plotted straight up at +90°. The capacitor current vector of 12 amperes is attached to the end of the RL circuit current vector. The arrow from the origin to the point where the capacitor current vector ends the sum of the current of the RL branch (18.5 A) plus the current through the capacitor branch (12 A).

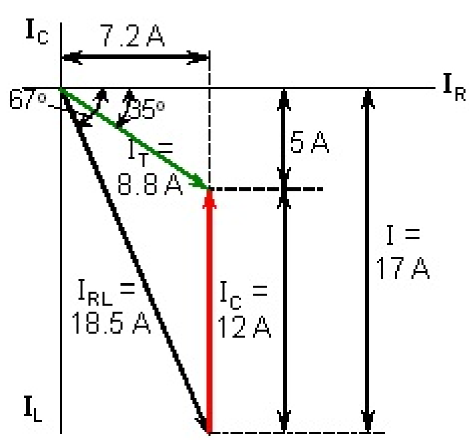

Figure 222.8 *The new total current of the parallel circuit (8.8 A) is the vector from the origin to the tip of the capacitor current vector. The new total current forms the hypotenuse of a right triangle. The calculations are shown below.*

To determine the new total current by this graphical method some trigonometry is required. First look at *Figure 222.6* for the vector of the current of the RL circuit which forms the hypotenuse of a right triangle. The first task is the find the value of the two sides of that triangle and the angle shown in *Figure 222.9.*

$$Vertical = 18.5 \times Sin67^{\circ} = 18.5 \times 0.92 = 17\,A$$

$$Horizontal = 18.5 \times Cos67^{\circ} = 18.5 \times 0.39 = 7.2\,A$$

Look at *Figure 222.9* and see that the horizontal side of the right triangle is 7.2 A and the vertical side is 5 A. Now it is possible to calculate the length of the hypotenuse of the triangle which is the sum of the currents of the parallel circuit shown in *Figure 222.8.* **The new power factor** is the Cosine of the new angle of this right triangle. The **new power factor** (pf) value is **0.82.**

$$I_T = \sqrt{7.2^2 + 5^2} = \sqrt{76.84} = 8.8\,A$$

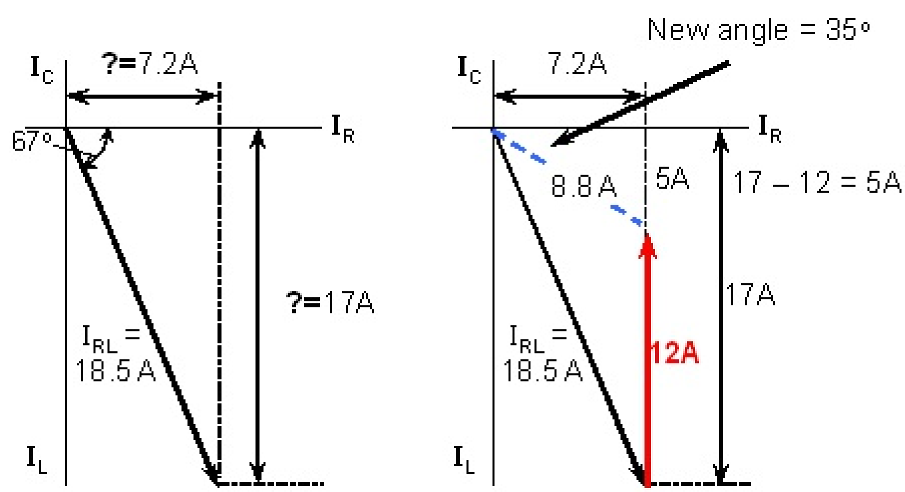

Figure 222.9 *This diagram shows how to determine the hypotenuse of the new right triangle which is the value of the total current of the circuit with the capacitor connected parallel to the RL circuit.*

$$\theta = Tan\frac{5}{7.2} = 35^{\circ} \qquad PowerFactor = Cos\theta = Cos35^{\circ} = 0.82$$