Electrical Tech Note — 205

Biosystems & Agricultural Engineering Department
Michigan State University

Alternating Current

Alternating current is produced commercially be spinning a magnetic field inside of a set of coils of wire. When there is motion between a wire and a magnetic flux such that the wire is cutting across perpendicular to the orientation of the magnetic flux, a voltage is induced into the wire. Since the magnetic field has two poles, a North and South, and is spinning, the stationary coils of wire are alternately exposed to a North pole and a South pole. The polarity of the voltage induced into the coil of wire will reverse each half revolution of the magnetic field. Over time the voltage will build up to a maximum then decrease to zero completing half a revolution of the magnetic filed. Then the polarity will reverse and the voltage will again build up to a maximum then decrease to zero. Viewed over time the voltage will trace out a sine wave each revolution of the magnetic flux. The frequency of the sine wave will be the revolutions per second of the magnetic flux inside the generator. Commercially available electrical power in the United States has a frequency of 60 Hz, which means the generator magnetic field is spinning at a rate of 3,600 revolutions per minute. The purpose of this Tech Note is to review the basic form in which alternating current is represented mathematically as a sine wave function.

Radians, Degrees, and Angular Velocity: The circumference of a circle is πd or $2\pi r$. If r = 1, then the circumference of a circle is πd. The orientation of the vector "r" can be specified at any time in terms of the distance traversed around a circle such as $\pi/4$ which is +45°. Rather than specifying the position of a vector about a circle in degrees, position can be specified in terms of radians where a complete circle is 2π radians. One radian is actually 57.3°. This concept is summarized in Figure 205.1.

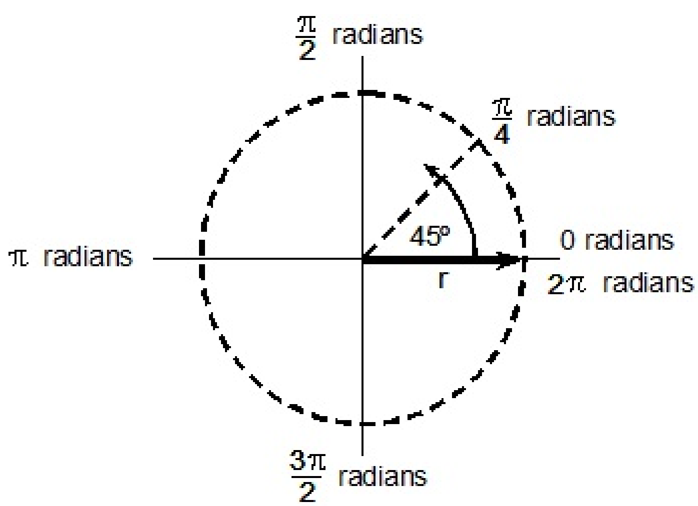

Figure 205.1 *Position of a vector (radius) about a circle can be specified in degrees or in radians where a complete circle is 2π radians.*

A sine wave can be generated by viewing the position of the tip of a vector representing the radius of a circle as it spins at a constant speed over a period of time. Refer to Figure 205.1 and imagine the center of the circle moving to the right as time passes while the vector is spinning in a counter-clockwise direction. A sine wave will be generated as shown in Figure 205.2. The rate at which the vector is spinning is the angular velocity (ω) which is generally specified in radians per second. Frequency of a sine wave is actually equivalent to the revolutions per second the vector is spinning to generate the sine wave. Since a complete circle is 2π radians, dividing the angular velocity (ω) by 2π radians will give the frequency (f) in cycles per second or Hz. Angular velocity (ω) is equal to 2π f. It is customary to represent the sine wave in the form of a cosine function as shown in Figure 205.2 where an initial angle in radians is introduced to shift the cosine function so that it starts at zero time at a value of zero.

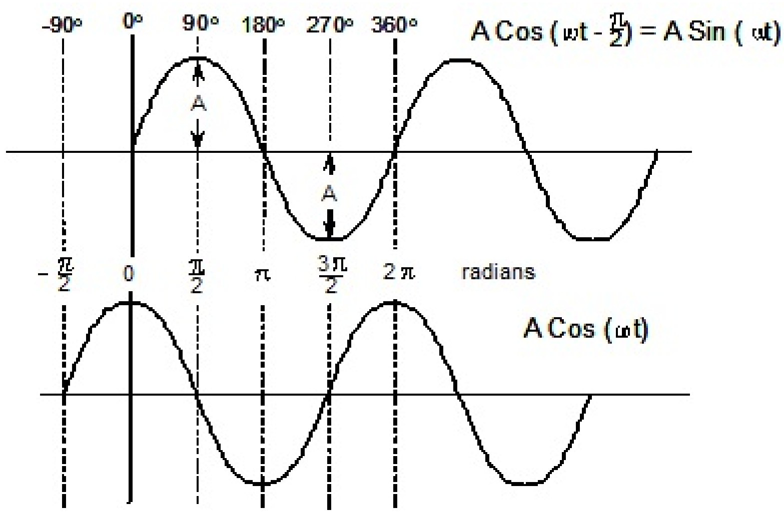

Figure 205.2 A sine wave can be represented with a cosine function by introducing a phase shift or angle shift so that the function at time zero starts at zero rather than the value one.

Effective or rms Value of a Sine Wave: Assume a voltage sine wave can be described with the function $v(t) = 170 \text{ Cos } (377t - 1.57)$. The value 170 is the positive and negative peak of the voltage wave. But this is not the steady state value of the voltage that can be used to determine desired steady state quantities of the circuit. The value that would be used in an Ohm's law calculation or a power calculation is called the "effective" value or "root mean square" (rms) value of the voltage. For a perfect sine wave or cosine wave, the ratio of the peak value to the effective value is the square root of two (1.414). If the effective value of the function $v(t) = 170 \text{ Cos } (377t - 1.57)$ is desired, divide 170 by 1.414 which will result in an effective value of 120 volts. This is illustrated in Figure 205.3.

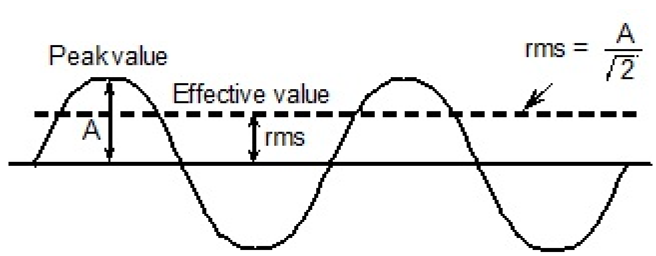

Figure 205.1 *Divide the peak value by 1.414 to get the effective (rms) value of the sine wave.*

Setting the Initial Starting Point of the Cosine Function: For this discussion the voltage function v(t) = 170 Cos (377t - 1.57) will be used. A cosine function at time zero and angle zero will have a value of 1. The function will appear like the top function in Figure 205.4. In order for the function to have a value of zero at time zero and appear as though it is a sine wave is to set the function starting point at -90° which is -1.57 radians. Compare the two cosine functions in Figure 205.4 with and without the initial starting point set at -1.57 radians.

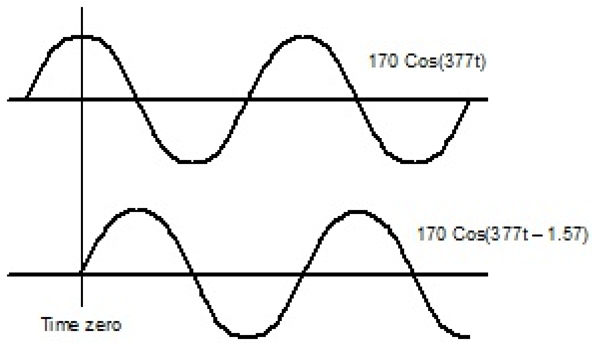

Figure 205.4 *Compare the trace of the cosine function with the starting point at 0 radians (top trace) with the bottom trace where the cosine function has a starting point at -1.57 radians.*

Example: For the following example use the voltage function v(t) = 170 Cos (377t - 1.57).

Determine the peak voltage of the waveform. **170 volts**

Determine the effective (rms) value of the voltage.
Divide the peak voltage by the square root of two (1.414) **120 volts**

Determine the frequency of the function.
The value 377 is two time pi times frequency.
So divide 377 by two pi. **60 Hz**