Electrical Tech Note — 502

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Amplification and Gain

An input electrical signal from a source often is too weak to perform the task desired. An amplifier is used to increase signal strength. Amplification is the process where gain is the result. Sometimes an amplifier is used as a buffer to prevent one portion of a circuit from having an adverse effect on another part of a circuit. Amplifiers have limitations and depending upon the task to be accomplished, the desired result may be accomplished with a single amplifier, or a series of amplifiers may be required. Typically amplification is intended to increase the power or voltage of an input signal. Changing the current level of an input signal may also be the primary objective, or changing the resistance level between two portions of a circuit. All of these quantities are related through Ohm's law and the power formula. It is the purpose of this discussion to explain the basic terminology and principles of amplification and not the methodology of how specific amplifiers operate. Those topics will be discussed in other Tech Notes. Understanding logarithms and exponents is necessary for the understanding of electrical signal amplification. Exponents and logarithms are covered in Tech Note 501.

Power Gain: The power gain of an amplifier is the ratio of the output power divided by the input power. Assume for example that a signal source can only provide 0.1 mW (milli watts) of power. Assume also that the output of this amplifier is 200 mW. The power gain is 200 mW divided by 0.1 mW which gives 2000. The symbol used for power gain is frequently the letter G or A_P. The gain of this amplifier is 2000. Using the letter P as the symbol for power, the formula for power gain G of an amplifier is given by Equation 502.1. As you will soon find out, gain can be represented in different ways and that can be very confusing.

$$G \text{ or } A_P = \frac{Po}{Pi} \qquad \textbf{Equation 502.1}$$

(Where Po is output power and Pi is input power)

Gain given as a ratio of output power divided by input power is easy to understand. When related to amplification of an audio signal intended for human listening gain represented as a ratio of output to input power is not very useful. The human ear does not respond to an increase in audio signal power in a linear manner. In order for the average human ear to detect a difference in audio level, one signal must have a power level 26% higher than the other signal. This is illustrated in Figure 1 where each number on the horizontal axis represents the ability to distinguish a difference in audio level from the adjacent number. The vertical axis is the normalized power level required to achieve a step change in audio level perception. Normalizing means that all power levels are divided by the first power level so that the first power level is 1.0 and all other power levels are a percentage of the first level. Only ten values are plotted in Figure 1 which shows that the relationship is exponential. The straight line on the same graph is the common logarithm of the normalized power level multiplied by ten. By taking the common logarithm of the values on the exponential curve, the curve is converted to a straight line. The straight line is the power level in decibels (dB). Note from Figure 1 that in order for the average human ear to detect a change in sound level, a 1dB change in level must occur. Gain, especially with audio systems, is expressed in decibels (dB) rather than an actual ratio of output to input power because there is a linear relationship between decibel level and the human ear response to sound. There are other reasons why gain is expressed in decibels and those reasons will also be discussed later.

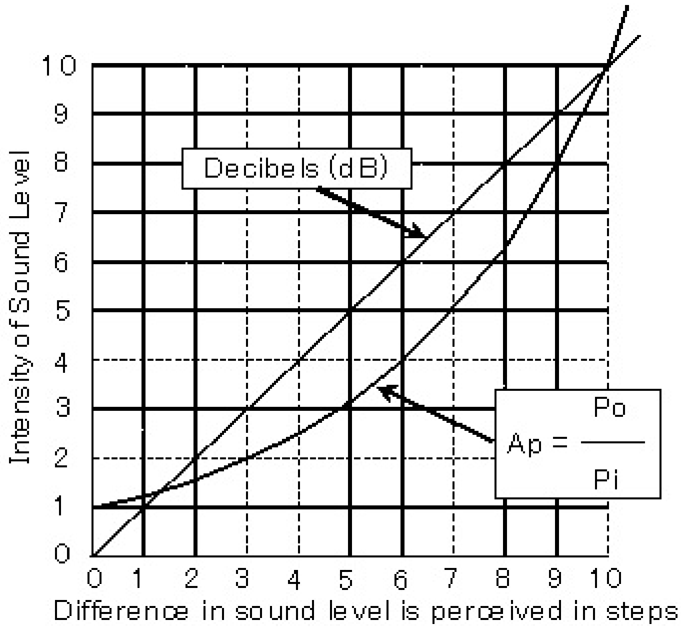
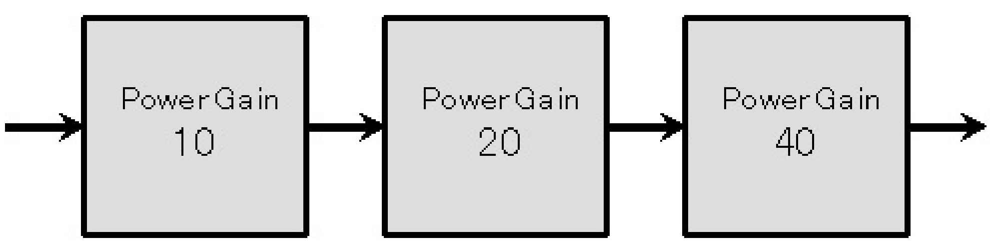


Figure 1. *Each mark on the horizontal axis represents the ability of the average human ear to perceive a difference in sound level while the vertical axis represents normalized ratio of the sound level as well as the decibel (dB) level.*

Cascade Amplifier Stages: Amplifiers are frequently connected in series or cascaded as shown in Figure 2. Each amplifier boosts the input signal by the gain that can be achieved by that amplifier stage. Using Equation 502.1, the output power of the first stage is P_{O1} which is the product of P_{i1} and the gain G_1 ($P_{O1} = P_{i1} \times G_1$). The output of the first amplifier stage is multiplied by the gain of the second stage to obtain the output of the second stage ($P_{O2} = P_{O1} \times G_2$). This process continues for as many stages there are connected in series. The overall gain of the amplifier stages is simply the product of the gain of all the stages ($G_T = G_1 \times G_2 \times G_3$). To get the output power directly, multiply the input power by the overall gain ($P_O = P_i \times G_T$).



$$10 \times 20 \times 40 = 8000$$

Figure 2. *An amplifier consists of three cascade stages with gains of +10, +20, and +40 and has an overall total gain of 8,000.*

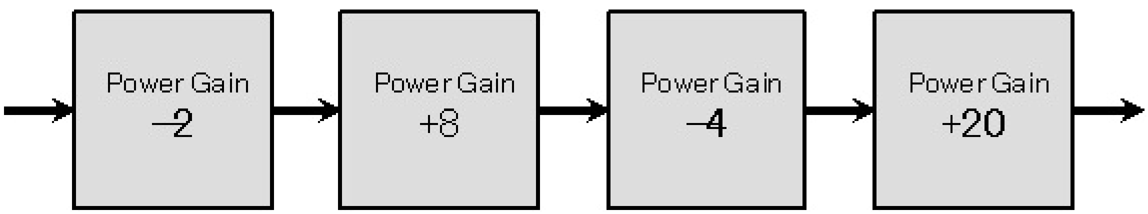
Passive components in a system can attenuate a signal, thus introducing a negative gain. The resistance of a length of connecting cable can introduce a negative gain. It is not uncommon for a system to contain a series of components some of which have a positive gain and some with negative gain. If the gain is negative, then divide by the value of the gain or multiply by the reciprocal of the negative gain.

$$P_O = P_i \times \frac{1}{G} = \frac{P_i}{G} \qquad \text{where gain is negative}$$

The example of Figure 3 consists of several components some of which have negative gains and some have positive gains. To determine the total gain of the system, multiply the positive gains and divide by the negative gains as shown in Equation 502.2.

$$\mathbf{P_O = P_i \times \frac{G_{1P} \times G_{2P}}{G_{1N} \times G_{2N}}} \qquad \textbf{Equation 502.2}$$

where P designates positive gain and N designates negative gain



$$\text{Gain} = \frac{8 \times 20}{2 \times 4} = 20$$

Figure 3. *Two components of the system have negative power gains of* – *2 and* – *4 and two components have positive power gains of +8 and +10, and has an overall total gain of 20.*

Gain Expressed in Decibels (dB): Referring to Figure 1, gain can be represented as a ratio of actual power level or it can be represented as the logarithm of the ratio of power level. Because there is a linear relationship between the logarithm of the ratio of power level and human hearing perception, it was customary in early days of electronics to rate gain in decibels rather than actual power ratios. It is common to rate power levels for electronic components in the form of decibels (dB) rather than actual values of power or even power ratios. The person working with electronics must be able to understand the meaning of power given in terms of decibels as well as be able to convert from decibels to power gain and voltage gain.

Power in bels (B) is the common logarithm of the power gain (ratio of output power divided by the input power). The bel is a rather large unit that can be converted to tents of a bell (decibel, dB) by multiplying by 10, Equation 502.3.

$$\textbf{bel power gain} = \log_{10} \frac{Po}{Pi} \qquad \textbf{Equation 502.2}$$

1 bel (B) = 10 decibels (dB)

$$\textbf{decibel power gain (dB)} = 10 \times \log_{10} \frac{Po}{Pi} \qquad \textbf{Equation 502.3}$$

There are some advantages to working with gain in decibels (dB) especially if the overall gain of several components connected in series is desired. First review the basic rules of multiplication and division for logarithms from Tech Note 501. Those rules are summarized as follows:

$$\text{Log } xy = \text{Log } x + \text{Log } y \qquad \text{Equation 501.3}$$

$$\text{Log } \frac{x}{y} = \text{Log } x - \text{Log } y \qquad \text{Equation 501.4}$$

Now recall from Equation 502.2 that total gain (G_T) is the product of the gain of the components with positive gain. If the logarithm of the product of two numbers is needed, all that is required is to add the individual logarithm of the two numbers (Equation 501.3). Figure 4 is the same system of amplifiers as shown in Figure 2 except the gain is given in decibels (dB) rather than as a ratio of output over input power. To get the total gain in dB simply add the dB gain of each component connected in series. For the system of amplifiers shown in Figure 4, the gain is 39dB.

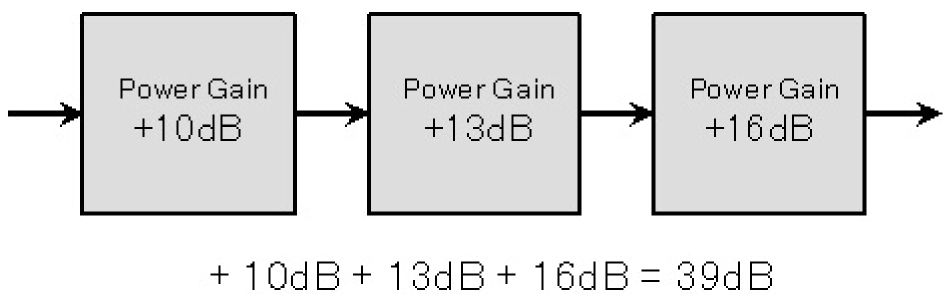
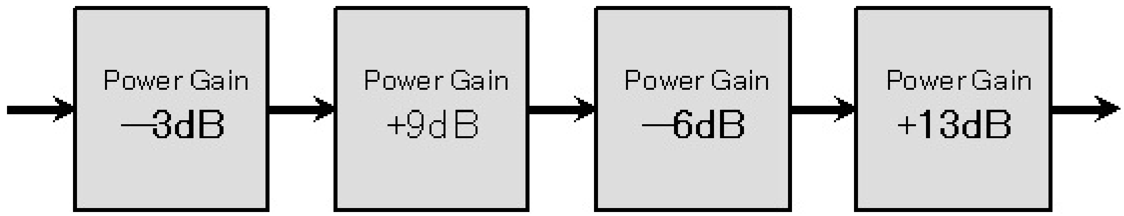


Figure 4*. An amplifier consists of three cascade stages with gains of +10dB, +13dB, and +16dB and has an overall total gain of 39dB.*

Refer to the Equation 502.2 for total gain and note that if the gain of the individual component is negative that value is divided into the gain of the system. Note from Equation 501.4 that when taking the logarithm of two numbers one of which is divided by the other, it is permitted to subtract the logarithm of the denominator from the logarithm of the numerator. This makes it easy to determine the total gain of a system when the gain of each component connected in series is provided in units of decibels (dB). All that is required is to take the arithmetic sum of the gains of the individual components. This is illustrated in Figure 5 where the gains of the individual components are -3dB, +9dB, -6dB, and +13dB. The actual amplification of the system is 13dB. Figure 5 is the same system of components as Figure 3, therefore, 13dB must be equal to a power gain ratio of 20. Figure 4 is the same system of components as Figure 2, therefore, 39dB must be equal to a power gain ratio of 8000. This conversion can be done on a calculator through the use of logarithms and exponents.



– 3d

B + 9dB – 6dB + 13dB = +13dB

Figure 5. *Two components of the system have negative power gains of −3dB and −6dB and two components have positive power gains of +9dB and +13dB, and has an overall total gain of +13dB.*

Converting Decibels (dB) to Gain as a Ratio of Output to Input Power: Keep in mind that power gain (ratio of output power divided by input power) may be given by the symbol G or it may be given by the symbol Ap. For this discussion the symbol G will be used. It may be helpful to refer to Tech Note 501 discussing logarithms and exponents. The conversion from power gain to dB gain is accomplished using Equation 502.4, and the conversion from dB gain to power gain can be made using Equation 502.5.

$$G = \frac{Po}{Pi} \qquad B = \frac{dB}{10}$$

$$\mathbf{dB = 10 \times \log_{10} G} \qquad \textbf{Equation 502.4}$$

$$\mathbf{G = 10^{(dB/10)}} \qquad \textbf{Equation 502.5}$$

For the examples of Figure 2 through Figure 5, the conversions from dB gain to power gain are as follows: (the following conversions are rounded off and are not exact) Remember that the log of the power gain is in bels and must be converted to decibels. To convert from decibels to power gain the value in decibels must first be converted to bels before it is used as an exponent of 10.

$$dB = 10 \times \log_{10} 8000 \approx 10 \times 3.9 \approx 39dB$$

$$dB = 10 \times \log_{10} 20 \approx 10 \times 1.3 \approx 13dB$$

$$G = 10^{3.9} \approx 8000 \qquad (39dB/10) = 3.9B$$

$$G = 10^{1.3} \approx 20 \qquad (13dB/10) = 1.3B$$

Voltage Gain (Av): For many electronic applications, voltage gain is more important a parameter than power gain. Voltage gain is also easier to determine from a circuit than power gain. Power is the product of voltage and current. Substituting Ohm's law into the power formula to eliminate current and power is the voltage squared divided by resistance.

$$\text{Power} = \frac{V^2}{R} \quad \text{or} \quad (V^2/R)$$

It is customary when making reference to gain (a ratio of output divided by input) to consider the gain as a ratio of power. Keeping with this tradition, gain is generally given as the ratio of power or as the ratio of the square of the voltage. Sometimes it is stated that a value is voltage gain and in those cases gain is simply the ratio of the output voltage divided by the input voltage. You must understand there is a difference, and be alert when reviewing data about gain diagrams and values provided for electronic components.

The following formulas show how power gain can be determined in terms of input and output voltage and input and output resistance. The second formula shows that in the spacial case where the input resistance and the output resistance are equal, the resistance cancels out of the formula and the power gain is simply the ratio of the square of the output voltage divided by the input voltage. This is the **impedance matched case** and it does occur especially with communications equipment.

$$\text{Power Gain} = \frac{Po}{Pi} = \frac{(V_O^2/R_O)}{(V_i^2/R_i)}$$

$$\text{Power Gain} = \frac{Po}{Pi} = \frac{(V_O^2/R_O)}{(V_i^2/R_O)} = \frac{V_O^2}{V_i^2} \qquad \text{where } R_O = R_i$$

When voltage gain is converted to decibels, it is customary to express the voltage gain as the logarithm of the square of the ratio of output voltage divided by the input voltage. By doing this the voltage gain and the power gain will be equal for the impedance matched case where the input and output resistance is equal. At this point it is necessary to refer back to Tech Note 501 for the rule dealing with the logarithm of a number raised to a power, Equation 501.5.

$$\text{Log } y^n = n \times \text{Log } y \qquad \text{Equation 501.5}$$

The logarithm of the ratio of the **square** of the output over input voltage is equal to **two times** the logarithm of the ratio of output over input voltage. This will give the voltage gain in bels. To convert to decibels (dB) multiply by ten, Equation 502.3. Decibel voltage gain is expressed as 20 times the logarithm of the ratio of output voltage divided by the input voltage as shown in Equation 502.6. In the special case where the input resistance is equal to the output resistance, decibel power gain is equal to the decibel voltage gain. In all other cases these two values of decibel gain are **not** equal.

$$\textbf{decibel voltage gain (dB)} = 20 \times \log_{10} \frac{Vo}{Vi} \qquad \textbf{Equation 502.6}$$

Other dB Terminology: Decibel values by themselves are ratios of change converted to a logarithmic scale. Sometimes it is beneficial to use the decibel scale, but with relationship to a specific value. The designation dBm is just one example. The letter **m** designates power in milliwatts where one milliwatt is the base value. If the input and output power are both 1mW then the ratio of output over input power will be 1 and the logarithm of that value will be zero. If you refer to Figure 1 of Tech Note 501, you will notice that when the ratio is less than one the logarithm is a negative number. If a standard signal power is 1mW, the signal strength can be adjusted up or down by turning a dBm knob. If the knob is turned up to +3 dBm the power will be doubled, and if the knob is turned to −3 dBm the power will be cut in half. Another common designation is dBmv which simply means the standard value is one millivolt.