Electrical Tech Note — 513

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Inductors[1]

Inductors are frequently used components in electronic circuits. An inductor consists of a coil of wire through which current flows. Current flowing in a conductor produces a magnetic flux around the conductor. When the magnitude of the current is changing the magnetic flux around the conductor is in motion. Another principle of magnetism is that when a wire experiences a moving magnetic flux, a voltage will be produced in the wire. If the moving magnetic flux is caused by a change in current flow in a conductor, the voltage produced in the conductor by the moving magnetic flux will oppose the change in current flow. This opposition to the change in magnitude of current flow is called *inductance*. *Inductance* opposes a change in current. Inductance has many applications in electrical circuits. The property of *self induction* is the opposition to a change in current flow in a conductor, and *mutual induction* is where the magnetic flux produced by one conductor produces a voltage and resulting current flow in another conductor. This Tech Note will discuss the parameters that determine inductance, different types of inductors, specifications, connecting them in series and parallel, and how inductors interact with resistance in a circuit.

Characteristics of an Inductor: An inductor consists of a wire coiled around a core material. An inductor is a passive device that stores energy in the form of a magnetic flux around the coil. This magnetic flux remains stationary in place as long as there is a constant flow of current. If the current changes in magnitude the magnetic flux will move as it stores even more energy due to increasing current or releases energy due to a reduction in current. Forming a wire into a coil such as shown in Figure 513.1 results in concentration of the magnetic flux produced by each turn of wire in the coil. *Inductance* is the opposition to a change in current, the unit of inductance is the *Henry*, and the symbol for inductance is the letter **L**. More about magnetism and inductance can be found in Tech Notes 217 and 317.

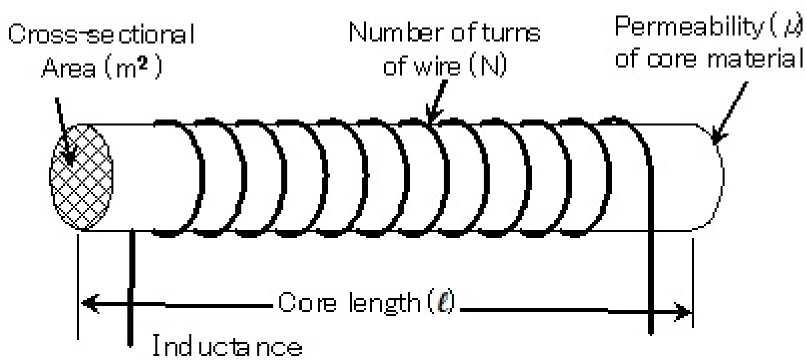

Figure 513.1 *An inductor consists of a coil of wire around a core material where the inductance is the opposition to a change in rate of current flow through the coil.*

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The factors that determine the inductance of a coil are the number of turns of wire in the coil (N), the cross-sectional area of the coil (A), the length of the core (l), and the permeability of the core (μ). Permeability quantifies the quality of a material as a path for a magnetic flux. Air and a vacuum have low permeability while metals containing iron have high permeability. The opposite of permeability is reluctance. The inductance of the coil is equal to the core permeability times the square of the number of turns of wire times the cross-sectional area of the core, divided by the length of the core (Equation 513.1). The fundamental core material is air or a vacuum and the permeability is 1.26×10^{-6} Henry per meter. Other materiels are described by a *relative permeability* (μ_r) which is compared to the permeability of air (μ_o). Any material containing iron is a good conductive path for a magnetic flux and has a high relative permeability. The permeability of an iron core can be several thousand compared to air. One way an inductor is used in a sensor is to have some measurand cause a change in the inductance of a coil. The parameter that is easiest to change is the permeability of the core. An iron or steel core can be physically moved into or out-of the coil resulting is significant changes in the inductance of the coil.

$$\textbf{Inductance (L)} = \frac{N^2 \times A \times \mu_r \times (1.26 \times 10^{-6})}{\text{core length } (l)} \qquad \textbf{Equation 513.1}$$

N	number of turns of wire in the coil
A	cross-sectional area of the coil (m^2)
μ_r	relative permeability of core material
1.26×10^{-6}	permeability of air or vacuum (Henry/m)
l	length of core (m)

Specifications: Since an inductor is surrounded by a magnetic flux when current is flowing through the inductor, this magnetic flux may have an influence on adjacent components of the circuit. In those cases where errant magnetic flux may be a problem the inductor can be enclosed in an aluminum or copper case. The copper or aluminum will act as a shield by intercepting the flux and converting the energy to eddy currents in the metal.

Inductors are made for specific applications. For many applications the power rating is small and generally not a problem. For applications, such as use in a dc power supply, the power rating of the inductor must be considered. The insulation on the wires in an inductor is a very thin film. Varnish has been used as insulation on inductor wire. In order to achieve the desired level of inductance in a small space the wire insulation must be thin. The limitation on the power rating of an inductor is it's ability to dissipate the heat produced within the inductor due to the resistance of the wire. The power produced by an inductor is equal to the square of the current times the resistance of the windings. The resistance of the windings can be measured with an ohmmeter, although when the wire temperature rises during use the resistance will also rise.

The permeability of the core material has a significant effect on the performance of an inductor. Many inductors have air cores. The permeability of air is quite low at 1.26×10^{-6} Henry/m. For radio frequency applications the core is often a powdered iron slug that usually has an adjustable position. Another common type of core is ferrite which is a ceramic material with magnetic properties. For power applications the inductor is wound on a laminated steel core.

Inductors in Series and Parallel: Inductors can be combined in series and in parallel if necessary to obtain the desired inductance. Increasing the number of turns of wire on the inductor will increase the inductance according to Equation 513.1. Connecting inductors in series, as shown in Figure 513.2, is like increasing the number of turns of wire. The total inductance is the sum of the values of the inductors connected in series as in Equation 513.2. All inductors will have resistance since they are made from a long length of wire wrapped into a coil. Increasing the inductance by increasing the number of turns will also increase the resistance of the inductor. For many applications inductor resistance is insignificant, but in some situations inductor resistance may have a significant effect upon the circuit.

$L_T = L_1 + L_2 + L_3$ **Equation 513.2**

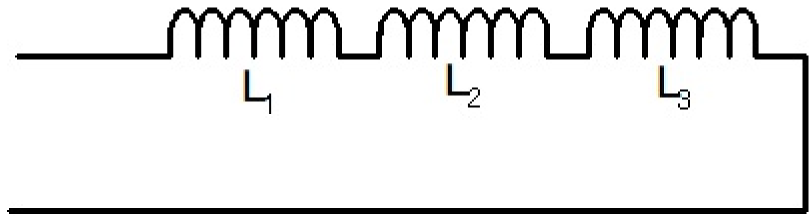

Figure 513.2 *When inductors are connected in series the resultant inductance is equal to the sum of the inductance of the individual inductors.*

Connecting inductors in parallel, as shown in Figure 513.3, will result in a dividing of the current such that less of the total current will pass through each inductor which will result in the final inductance being less than the smallest value of inductor connected in parallel. The reciprocal of the resulting inductance of several inductors connected in parallel is the sum of the reciprocals of the values of the individual paralleled inductors according to Equation 513.3. After the reciprocals have been summed, take the reciprocal of the final value to get the resultant inductance. If the inductors are all of the same value then all that is required is to divide the value of one inductor by the number of inductors arranged in parallel. For example if two 470 mH inductors are connected in parallel the resultant value will be 470 mH divided by 2 which is equal to 235 mH. Since an inductor has some resistance, which is not responsive to frequency, be careful when connecting inductors of unequal inductance in parallel. The unequal resistance of the inductors may have a significant effect on the division of current which can produce an unexpected result.

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$ **Equation 513.3**

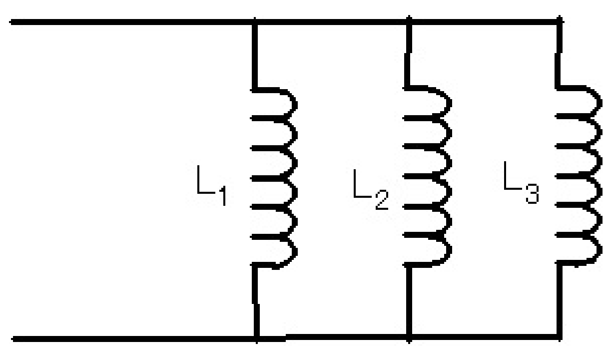

Figure 513.3 *When inductors are connected in parallel the reciprocal of the resultant inductance is equal to the sum of the reciprocal of the individual inductors connected in parallel.*

Inductive Reactance: In order to determine the effect of an inductor on a circuit it is necessary to convert the value of inductance into a form that is compatible with Ohm's law since an inductor has a resisting effect on the circuit. The effect of a specific inductor in a circuit depends upon the frequency of the electrical supply involved. The term used is *reactance* and in this case the quantity is *inductive reactance*. The unit of inductive reactance is the Ohm, but inductive reactance cannot be added arithmetically to resistance. Both are vectors and they act at a right angle to each other. The symbol for reactance is the letter **X** and in particular inductive reactance has a subscript so it is designated as $\mathbf{X_L}$. Equation 513.4 is used to determine the inductive reactance given the value of the inductor in Henry (L) and the frequency of the electrical supply (f) in Hertz.

$$X_L = 2\pi f L$$ **Equation 513.4**

The current limiting effect a particular inductor has on a circuit depends upon the frequency of the electrical supply. Note in Equation 513.4 that frequency in Hertz (f) is in the numerator which means that the higher the frequency the higher will be the inductive reactance for a given value of inductance. Here are three examples where a 0.47mH inductor is placed in a dc circuit (0 Hz), an ac circuit operating at 60 Hz, and an ac circuit operating at 200 kHz. This is summarized in Figure 513.4.

$L = 0.47\text{mH} = 0.00047\text{H} = 0.47 \times 10^{-3}\text{ H}$

where f = 0 Hz, $X_L = 0\ \Omega$

where f = 60 Hz, $X_L = 0.18\ \Omega$

where f = 200 kHz, $X_L = 591\ \Omega$

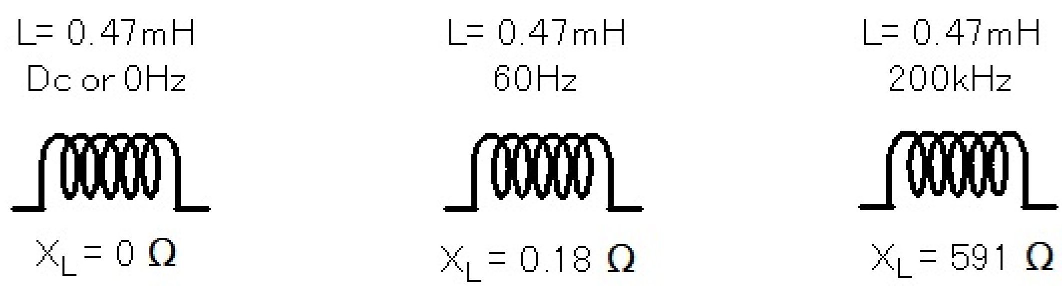

Figure 513.4 *Inductive reactance of a particular inductor increases as the frequency increases.*

In a direct current circuit with a constant voltage the inductor acts like a closed switch or short circuit offering no resistance to the flow of current. In a practical situation an ideal inductor does not exist and there is always some resistance in series with the inductance since the inductor is constructed from a long length of wire. This resistance, using Ohm's law, will limit the steady state current when the inductor is connected to a direct current source. At very high frequency the inductor can essentially block the flow of current in a circuit. Because the effect of an inductor is different depending upon the frequency, an inductor can be useful in filtering where multiple frequencies are present in a circuit. The value of inductance in combination with other components can be chosen to attenuate some frequencies and pass other frequencies. When an inductor is connected in series with a load resistor (R_L) as shown in Figure 513.5, it passes low frequencies and blocks high frequencies. This is called a low pass filter. When the inductor is placed in parallel with the load resistor (R_L), also shown in Figure 513.5, it shunts low frequencies past the load resistor so that only high frequencies are forced through the resistor to be detected as a voltage drop across the resistor. This is called a high pass filter. When examining Figure 513.5 keep in mind that in the practical case there is a small resistance in series with the inductance.

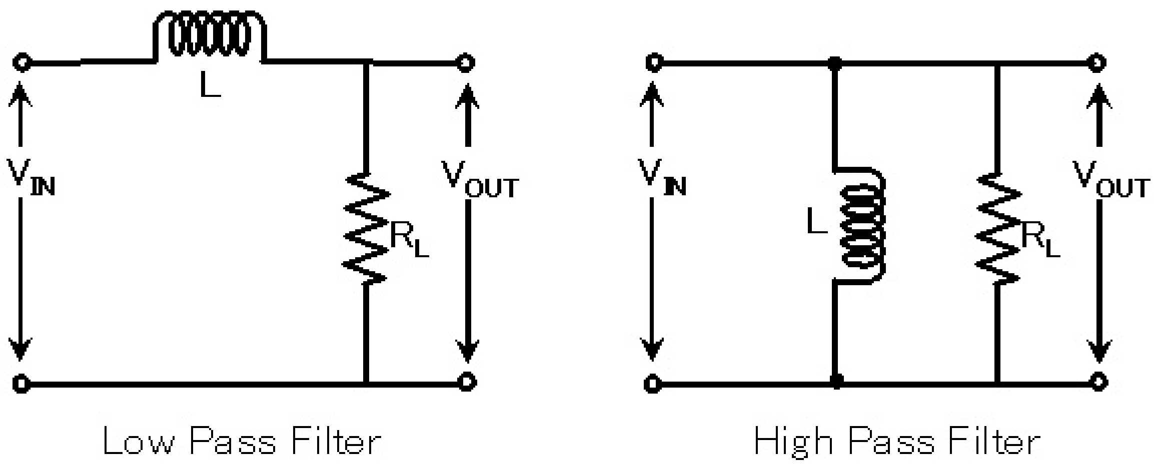

Figure 513.5 *An inductor can be used in a circuit with multiple frequencies to help select the desired frequencies and reject other frequencies. Connected in series with a load resistor it acts to pass low frequencies and when connected in parallel with the load resistor it acts to reject low frequencies.*

Impedance: In an alternating current circuit the value of inductive reactance (X_L) is determined using Equation 513.4. Consider an ideal inductor connected to a dc voltage source where there is no resistance to the circuit other than the internal resistance of the voltage source. This is illustrated in Figure 513.6. Since inductance opposes a change in current, the instant the switch is closed rate of change of current will be at a maximum and for that instant the current will be zero. At this instant when the switch is closed the voltage across the inductor coil (E_L) will be equal to the source voltage (E_S). Since inductance is opposing a change in current, obviously the current in the circuit increases after the switch is closed. The current will increase in an exponential manner until reaching a maximum which is determined using Ohm's law and dividing the source voltage (E_S) by the circuit resistance (R_S). The voltage across an ideal inductor (E_L) will be zero when the current reaches a maximum and is no longer changing.

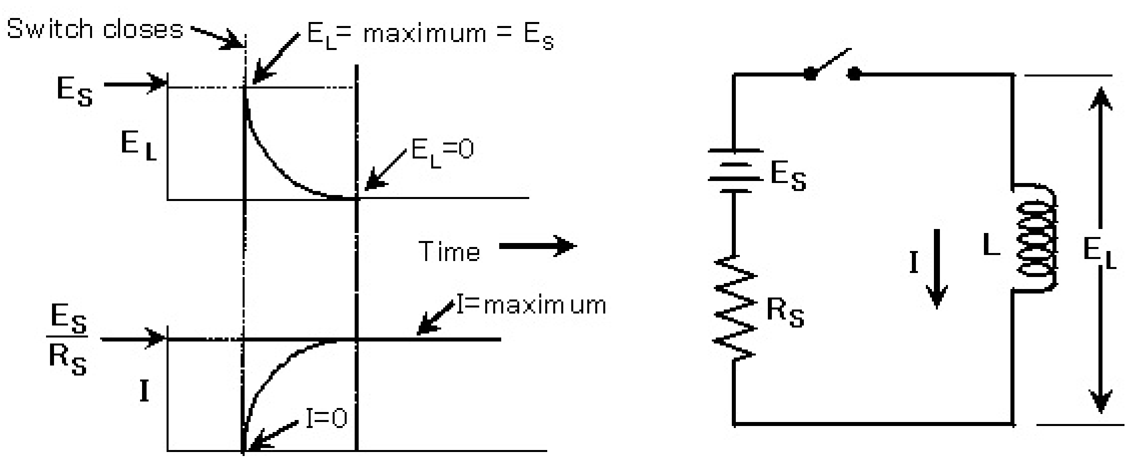

Figure 513.6 *When the switch is closed and a dc voltage is applied to the inductor, current flow will be zero then increase in an exponential manner eventually reaching a maximum. Voltage across the inductor initially matches the source then decreases in an exponential manner to zero.*

In the practical case there is some resistance to the inductor and there will be some voltage across the inductor. Note in Figure 513.6 that when the current is at zero the voltage across the inductance is at a maximum, and when the current reaches a maximum the voltage across the inductance is at zero. When the current attempts to change it meets with an opposing voltage that tries to prevent change. In an inductive circuit, voltage occurs first, then current eventually follows. This is not the case with a resistor. With a resistor, the voltage and current are zero at the same time and at a maximum at the same time. When the voltage source is a sine wave, with an ideal inductor the voltage and current sine waves are offset by 90°. With only a resistor in the circuit the voltage and current sine waves are exactly synchronized. The inductive reactance (X_L) as calculated using Equation 513.4, therefore, acts 90° out of synchronization with any series resistance in the circuit.

When an ac circuit contains both resistance (R) and inductive reactance (X_L) their combined effect is called *impedance* (Z), but even though they both have units in Ohms their values cannot simply be added to determine their combined effect. When a resistor and an inductor are connected in series in an ac circuit, an impedance diagram using vectors as resistance (R) and reactance (X_L and X_C) can be used to illustrate how resistance and reactance are combined to determine impedance (Z). Inductive reactance (X_L) occurs immediately when current tries to change then decreases over time as discussed earlier, therefore, in Figure 513.7 inductive reactance is 90° out-of-phase with resistance and leading the resistance. Inductive reactance (X_L) points straight up in the impedance diagram with resistance pointing to the right. Capacitive reactance (X_C) points straight down and is discussed in Tech Note 512. The resistance and inductive reactance form the two sides of a right triangle, as illustrated in Figure 513.7, and their net effect or impedance is the hypotenuse of that triangle. From mathematics it is known that the sum of the squares of the two sides of the right triangle is equal to the square of the hypotenuse. Impedance (Z) of an ac circuit can be determined using Equation 513.5 when the values of resistance and inductive reactance are connected in series. Equation 513.5 only works for the case where the resistance, inductive reactance, and capacitive reactance are connected in series. In a series circuit an inductor and a capacitor cancel the effect of the other and their combined effect is the difference between their reactance values. If the circuit does not contain a capacitor, the value of X_C is entered as zero.

$$Z = \sqrt{R^2 + (X_C - X_L)^2} \qquad \textbf{Equation 513.5}$$

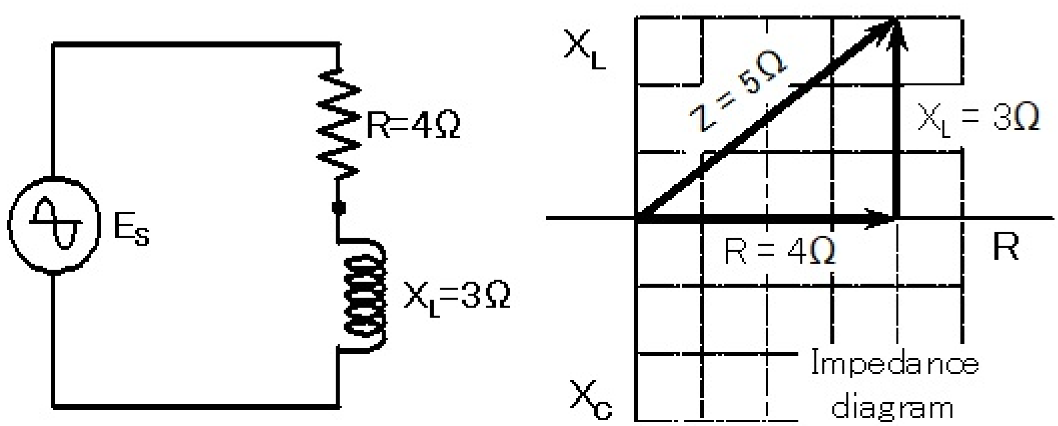

Figure 513.7 *When a resistor and an inductor are connected in series in an ac circuit the resistance and inductive reactance act 90° out-of-phase with each other with the inductive reactance pointing up in an impedance diagram and resistance pointing to the right. The hypotenuse of the right triangle that is formed is the value of their net effect or impedance.*

Induced Voltage of an Inductor: When a current flows in a wire a magnetic flux builds up around the wire with a strength that is proportional to the current. Another principle of electricity and magnetism is that when a wire is exposed to a moving magnetic flux a voltage is produced or induced into the wire. These principles are discussed in Tech Note 217. A change in level of current flowing in a wire will result in movement of the magnetic flux as the magnetic flux increases or decreases in strength. This movement of the magnetic flux induces a voltage in the wire that actually opposes the change in current. If the current tries to increase this induced voltage will oppose the increase, and if the current tries to decrease the induced voltage will attempt to maintain the current flow. This phenomena is called inductance, and the unit of measure is the Henry (L) which is actually the induced voltage divided by the time rate of change of current as in Equation 513.6. If the induced voltage due to a current change is measured or known and the rate of change of current is known in Amperes per second, the inductance (L) of the wire or coil of wire can be determined using Equation 513.6. By rearranging that equation the induced voltage can be determined from the inductance of a coil (L) and the rate of change of current (di/dt) using Equation 513.7. It is important to note that the magnitude of the induced voltage for a given inductance (L) is dependant upon the rate of change of current not the level of current. A high induced voltage can occur with a small current level change if that change occurs in a very short time interval.

$$L = \frac{E_L}{\frac{di}{dt}} \qquad \textbf{Equation 513.6}$$

$$E_L = L\frac{di}{dt} \qquad \textbf{Equation 513.7}$$

The concept of inductance in an electric circuit is somewhat like the inertia and momentum involved when moving a fluid through a system or moving an object across a surface where there is negligible friction. When a force is applied to move a heavy object the inertia of the object will resist movement at first and it will take a short time interval to get the object up to speed. Once the heavy object is in motion it has gained kinetic energy in the form of momentum. A considerable amount of force is required to get the heavy object in motion, but once the object is up to full speed no force is required to keep it moving provided there are no frictional forces present to slow it down. In the case of an electrical circuit when the switch is opened to terminate the current flow, the energy of the magnetic flux around the wire will be released. When the heavy moving object experiences a wall in it's path the energy of momentum must be dissipated. The momentum will attempt to keep the object moving even though the will is in the way. The result will most likely be damage to the object, the wall, or both as the energy is dissipated. When a switch is opened in a circuit containing an inductor, such as the circuit shown in Figure 513.6, the energy of the collapsing magnetic flux will attempt to keep the current flowing. An induced voltage will occur due to the decreasing current that will try to maintain current flow when the switch is opened. The amount of induced voltage will depend upon the inductance of the coil and the rate at which the current is decreasing in accordance with Equation 513.7. The current will decrease in an exponential manner as shown in Figure 513.8.

Assume for example that the current decreased in a linear manner from a level of 500mA to zero in a period of 20μs when a switch is opened. Also assume that the inductor in the circuit has a value of 20mH (L). Assuming a linear decrease in current the rate of current decrease will be 500mA divided by 20μs which is a rate of 0.025×10^6 Amperes per second (di/dt). Substituting these values for inductance (L) and rate of current decrease (di/dt) into Equation 513.7 gives a value of induced voltage (E_L) of 500 volts. This induced voltage is the source of arcing at the switch when it opens. In the case of a solid state switch, such as a transistor, this induces voltage that occurs when current is terminated will most likely result in damage to the solid state switch.

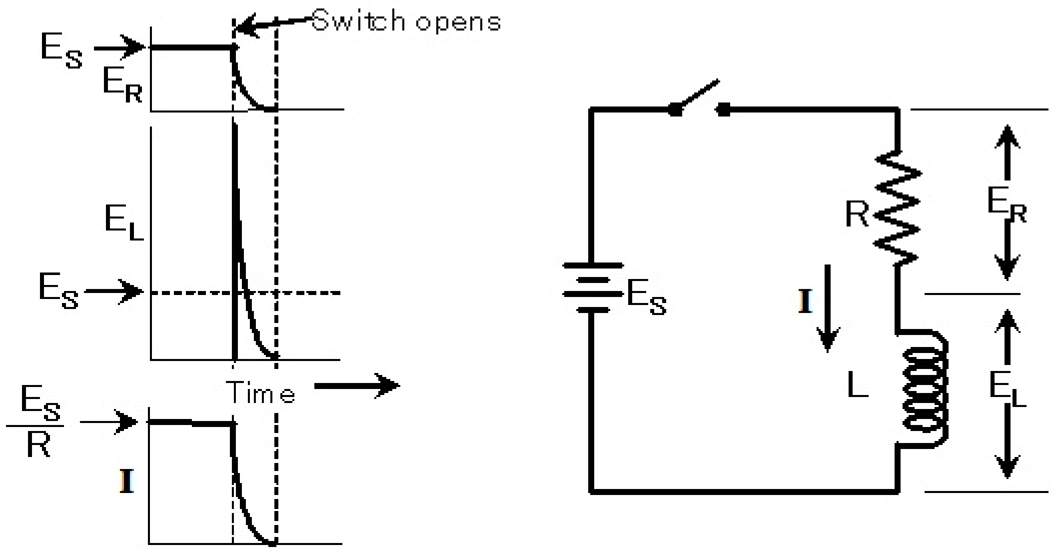

Figure 513.8 *When a switch is opened for a dc circuit containing an inductor in series with a resistance, the collapsing magnetic flux at the inductor will induce a voltage proportional to the rate of decrease of current that will attempt to maintain the current flow.*

In Figure 513.8 the inductor is represented by a separate resistor and a separate ideal inductor. In the real world the inductance and resistance cannot be separated and the actual voltage across the inductor is the sum of the two voltages represented in Figure 513.8. Note that the voltage across the resistor simply decays to zero in an exponential manner as the current decreases. The voltage across the inductor caused by the fast time rate of decay of current results in an induced transient voltage spike that is added to the voltage across the resistor, and at the instant the switch is opened the voltage across the switch contacts exceeds the supply voltage.

Time Response to Step Voltage Change: An inductor always consists of some resistance in series with the inductance by the nature of it's construction although this resistance may be negligible with respect to other resistance in the circuit. When an inductor in series with a resistor is energized with a fixed value of voltage, at first there will be no current flow and eventually the current will reach a maximum value that is equal to the supply voltage (E_S) divided by the value of the resistor (R). The current will increase in an exponential relationship as shown in Figure 513.6, and described by Equation 513.8. When the circuit is de-energized the current will decrease to zero also based upon the relationship of Equation 513.8 and shown in Figure 513.8. The maximum current is determined by Ohm's law which is the supply voltage (E_S) divided by the series resistance (R). Note in Equation 513.8 that the exponent of the value *e* (2.718) is the elapsed time in seconds multiplied by the series resistance (R) and divided by the inductance (L). Since the exponent has no dimension, inductance divided by resistance (L/R) must have the dimension of seconds (Joule second^2 per Coulomb^2 divided by Joule second per Coulomb^2 = seconds). Note that when the value of *t* is equal to the ratio of inductance over resistance (L/R) the value of the exponent of *e* is 1.0 and the value is 2.718. Subtracting the reciprocal of *e* from 1 in Equation 513.8 gives a value of 0.632. This means that when the exponent of *e* is 1 the current (i_L) has reached 63.2% of it's maximum level in a circuit consisting of an inductor in series with a resistor. For a circuit with an inductor connected in series with a resistor the value of inductance in Henry (L) divided by the resistance in Ohm (R) is called the time constant (τ), Equation 513.9. When five time constants have elapsed the current in the circuit has reached 99.4% of it's maximum level. The current verses time relationship for a series inductor and resistor circuit is shown in Figure 513.9. Many natural phenomenon behave like a series inductor-resistor circuit and exhibit this same exponential relationship. For these situations the time constant can be determined experimentally.

$$i_L = \frac{E_S}{R}\left(1 - \frac{1}{e^{\frac{Rt}{L}}}\right)$$ **Equation 513.8**

$$\tau = \frac{L}{R}$$ **Equation 513.9**

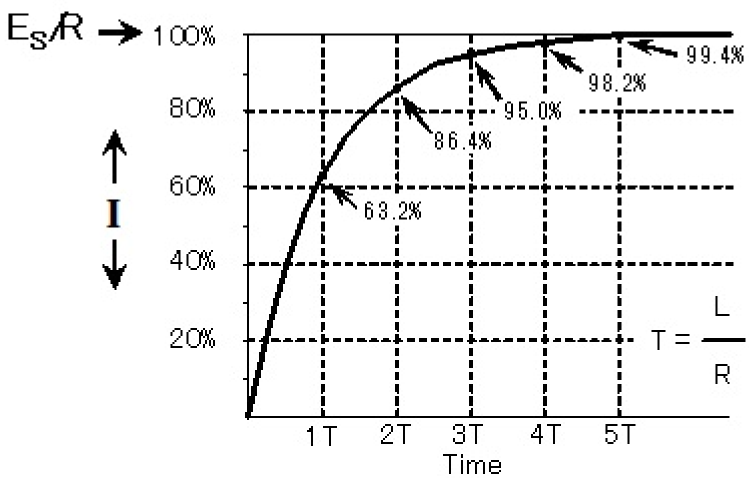

Figure 513.9 *When energized by a constant dc voltage, the current in a series inductor-resistor circuit increases according to a predictable exponential curve and reaches essentially maximum current in five time constants where the time constant is equal to the inductance divided by the series resistance.*

Energy Stored by Inductance: The inductance in Henry is the voltage that will be induced into the circuit by an inductor for a rate of change of current of one Ampere per second. Power at any point in time is the product of voltage and current. Substituting Equation 513.7 into the formula for power results in Equation 513.10 where the power at any point in time on an inductor is the product of the inductance (L), the current (i) and the time rate of change of current (di/dt). A dimensional analysis performed on Equation 513.10 results in Joule per second which is the Watt (Joule second^2/Coulonb^2 × Coulomb/second × Coulomb/second^2 = Joule/second = Watt). Note in Equation 513.10 that power is being expended in the circuit only when there is a time rate of change of current (di/dt). Examine Figure 513.6 and note that at the instant the circuit is energized the current (i) is zero and there is no power being delivered to the inductor. When the current reaches a maximum at the end of approximately five time constants the current is at a maximum, but the induced voltage is zero and no power is being delivered to the inductor. Between these two extremes there is a finite value of current and induced voltage, thus power is being delivered to the inductor to build a magnetic flux. Note also in Figure 513.8 that power is being returned to the circuit from the magnetic flux when the circuit is opened and there is a finite value of induced voltage and current flow for a brief time interval.

$$p = E_L i = Li \frac{di}{dt}$$ **Equation 513.10**

In order to determine the energy stored in a magnetic flux of an inductor it is necessary to integrate Equation 513.10 over the time interval the current is changing and obviously the relationship for the current must be known in order to perform the mathematics. For the case of the inductor energized with a fixed magnitude source as shown in Figure 513.6, integrating power with respect to time results in a simple relationship for the energy (Joule) stored in the magnetic flux of an inductor given the value of inductance (L), Equation 513.11.

$$W = \int pdt = \int Li \frac{di}{dt} dt = \int Lidi = \frac{1}{2} Li^2$$ **Equation 513.11**

Consider an example of a 200mH inductor energized with a 12 volt dc source where the total resistance of the circuit is 2 Ohm, and determine the energy stored in the magnetic field of the inductor. The energy is determined in this case using Equation 513.11 except it will first be required to determine the current. Ohm's law is used to determine the current which is 12 volt divided by 2 Ohm to give 6 Ampere. Finally put these values into Equation 513.11 to determine that 3.6 Joule of energy is stored in the magnetic flux. Remember that this energy will be returned to the circuit the instant the switch is opened to de-energize the circuit.

CAUTION: Relay contacts are designed to dissipate the energy released when the contacts open and relay solenoids are de-energized. A diode is often connected in parallel with a solenoid coil that is operated with a solid state switch. Collapsing magnetic flux from motors can add a significant current when a fault occurs in the wiring supplying motors.