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ELECTRICAL QUANTITIES

The basic terms used to quantify and describe electricity and the interactions of electrically charged bodies are discussed in this Tech Note. The familiar terms are explained in their basic form using the fundamental quantities of the meter (m) as distance, the **second (s) for time**, the **Coulomb (C) for a quantity of electrical charge, energy in Joules (J)**, and the kilogram (kg) for mass. Some facts about the structure of the atom are useful in gaining an understanding of the size and number of charged particles involved, and how these particles move to bring about the flow of electrical charge.

Electrical Charge: There are two distinct types of electrical charges, positive and negative. Charles Coulomb discovered that like charges repel each other and opposite charges attract. Charge, either positive or negative, is quantified in units called the Coulomb. One **Coulomb** is equal to 6,241,500,000,000,000,000 (6.2415×10^{18}) electrons or protons (6.2415 billion billion individual charges). The symbol used to represent electrical charge is the lower case letter **q**. A single electron or proton has a charge of 1.602×10^{-19} Coulomb. The force of attraction or repulsion between two objects each with a charge is proportional to the product of the charge of each object divided by the square of the distance between the two objects as illustrated in *Figure 211.1*.



Figure 211.1 The force of repulsion or attraction between two charges is equal to the product of the charges divided by the square of the distance between the charges.

Force is in Newtons (N), distance is in meters (m), and electrical charge is in Coulombs (C), the force that one electrically charged object has on another is determined using *Equation* 211.1. In order to solve the equation for actual charged objects, a constant number relating Coulombs, meters, and Newtons is required. This is called the *permittivity constant* (ε_0) and has the value 8.854 x 10⁻¹² C²/N•m².

The quantity of charge on each object such as shown in *Figure 211.1* is q_1 and q_2 . The distance from the center of one object to the center of the other object is d. *The force (F)* between the two objects will be in the SI metric term Newtons (N) where one **Newton** is equivalent to approximately 0.222 lbs force.

$$F = \frac{1}{4 \times \pi \times \varepsilon_o} \times \frac{q_1 \times q_2}{d^2}$$
 Eq. 211.1

Characteristics of a Metal Atom: All matter is composed of atoms which are building blocks composed of a nucleus with a positive charge and small particles with a negative charge in constant motion around the nucleus in spherical orbits. These particles in motion about the nucleus are called electrons and they are said to have a negative charge. The nucleus of most atoms is composed of two types of particles – protons and neutrons. The proton has a charge that is of equal magnitude but of opposite charge as an electron. The charge of the proton is said to be positive. The neutron does not have a charge. For every proton in the nucleus of an atom, there will be an electron moving in an orbit around the nucleus. A copper atom has 29 protons, 29 electrons and 35 neutrons. A copper atom actually has an electron moving in an orbit all by itself far from the nucleus and not held as tightly to the nucleus as the other electrons. Figure 211.2 is a representation of a copper atom. The outer electron shown in *Figure 211.2* is known as a **free electron** and due to thermal energy often drifts about randomly from atom to atom. Iron with 26 protons and 26 electrons has two electrons in an outer orbit far from the nucleus and not held as tightly to the nucleus as the other electrons. Having one or two loosely held outer electrons is a characteristic that makes a material a good conductor of electricity. These materials are all metals.





An electrically charged particle has mass and as a result will be attracted to another particle or object because of gravity. *The* **gravitational attraction** *is proportional to the mass of the particle and inversely proportional to the square of the distance between the centers of the objects.* In the case of electrically charged particles, the attraction of gravity is insignificant in most cases compared to the attraction or repulsion of the electrical charges. In most situations gravity can be ignored. The following is the approximate mass of the proton, neutron, and an electron:

Proton mass	=	1.673 x 10 ⁻²⁷ kg
Neutron mass	=	1.675 x 10 ⁻²⁷ kg
Electron mass	=	9.110 x 10 ⁻³¹ kg

The random movement of electrons in a solid metal such as copper from one atom to another is due to thermal energy. The net flow of electrical charge in any one direction is zero. There is approximately 10^{29} free electrons in one cubic meter of a metal conductor such as copper (10^{29} electrons/m³). *This is* 10^{20} electrons per cubic millimeter. To give some perspective to these numbers, the diameter of an atom including electrons ranges from approximately 10^{-13} m to 10^{-11} m. The diameter of the nucleus ranges from approximately 10^{-15} m to 10^{-14} m.

It is curious to note that with all of the positively charged protons of an atom crammed into such a small space as the nucleus, *there must be a repelling force between the protons*. Indeed this is the case and for the **iron atom** the protons in the nucleus are spaced apart from each other with an approximate average distance from center to center of 4×10^{-15} m. Using *Equation 211.1* the repelling force can be calculated. Charge of each proton is 1.602×10^{-19} C, and the *permittivity constant* (ϵ_0) is $8.854 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$. The *repelling force between two protons in the nucleus of an iron atom is estimated to be 14.4 Newtons which is equal to 3.2 pounds*. It is easy to understand why a huge amount of energy is produced by splitting the nucleus of an atom (fission) such as uranium 235 where some of this force is released as excess energy.

How Electrical Charge Flows: If a copper conductor is used to connect an object that has a negative charge (an excess of electrons) and a positive charge (a deficit of electrons), the loosely held free electrons in the copper will migrate in the direction towards the positive charge. If an electron leaves a copper atom, the atom no longer has a neutral charge – it has a positive charge. So when electrons (negative charges) move in one direction in a conductor, positive charges are left behind and appear to be moving in the opposite direction as illustrated in *Figure 211.3.* Any one electron actually moves quite slowly along the wire, but because one electron pushes on the one ahead of it and so on, the charges are actually moving through the conductor at nearly the speed of light with *negative charges moving in one direction and an equal number of positive charges moving in the opposite direction. Because both positive and negative charges are moving through the conductor, electrical current flow can be defined as charge moving from positive to negative or from negative to positive.*

Electrons will migrate in one direction when a conductor is connected to a source of electrical charge such as a battery. The free electrons when exposed to this electrical field will stop their random movements and flow in the direction of the electric field. *Electrons and their negative charge will move from the negative to the positive terminal of the battery, Positive charges will move from the positive to the negative to reminal of the battery.* Engineers and scientists define electrical current flow from positive to negative. This is called *conventional current flow* where the current is considered to flow from **positive to negative.** Many technical books define electrical current flow in the opposite direction or the direction of electron flow.



Figure 211.3 As electrons move in one direction through a conductor, a positive charge moves in the opposite direction.

Fundamental units of electricity are energy, charge, and time: The fundamental SI metric units that are used to define electricity are **charge** in Coulombs, **energy** in Joules, and **time** in seconds. The common units of electricity such as volts, amperes, ohms, and watts can be expressed using the following units:

energy		Joules
charge	—	Coulombs
time	_	seconds

Electrical current is defined as the rate of flow of charges through a conductor. Keep in mind that as a negative charge moves in one direction through a conductor, a positive charge of equal magnitude moves in the opposite direction through a conductor. Current flow is then charge per unit time or Coulombs per second. The unit of measure of electrical current is the *ampere which is equal to one Coulomb per second.*

1 Ampere = 1 Coulomb/second

Voltage is often defined as push or pressure causing the current to flow. Actually, **voltage** *is the energy level applied to each Coulomb of charge*. Current or charges flow out from a source through a circuit and back to the source. The energy in the charges gets extracted by the components of the circuit. **Voltage is energy per unit charge** or Joules per Coulomb.

Resistance is the opposition to electrical current flow. It is a property of the material through which the electrical current must flow. For example, copper offers less resistance to current flow than does aluminum. For most applications, resistance is a linear property of the material. This means that a conductor of a given cross-section will have twice as much resistance if the length is doubled. Resistance is inversely proportional to the cross-sectional area. If the cross-sectional area of a conductor is doubled, the resistance of that conductor of a given length will be cut in half. This does not necessarily mean, however, that doubling the cross-sectional area of a conductor will double the current carrying capacity of a conductor. There are other factors that affect the current carrying capacity of conductors. *The unit of*

measure of resistance is the **Ohm** and the symbol is the Greek letter omega (Ω). All materials exhibit a quantity called resistivity. This is a quantity that expresses how much resistance the material will offer to the flow of electrical charge. **Resistivity** is usually represented with the lower case Greek letter rho (ρ). When resistivity is given as the Greek letter rho (ρ), the units are usually in ohm-cm. Sometimes the capital letter *K* is used for resistivity. When the letter K is used the units of resistivity are usually ohm-circular mils per foot.

If the resistivity of a conductor is known the resistance of that conductor can be determined if the length and cross-sectional area are known. This works for any material not just metals. The relationship is expressed by *Equation 211.2.*

$$R = \frac{\rho \times L}{A}$$
Eq. 211.2a
$$R = \frac{K \times L}{A}$$
This is the equation used by electricians Eq. 211.2b

If \boldsymbol{p} is in ohm -cm, length L must be in centimeters (cm), and cross-sectional area A must be in square centimeters (cm²). If **K** is in ohm - circular mils per foot, length L must be in feet (ft), and cross-sectional area A must be in circular mils (cmil).

The following examples illustrate how *Equation 211.2* can be used to determine the electrical resistance through a material when the resistivity and dimensions are known. The first example involves a 2 in. diameter nonmetallic pipe that is 20 ft long and completely filled with chlorinated swimming pool water with a resistivity of approximately 300 Ω ·cm. The resistance of that column of pool water is 9022 Ohm. The length of the pipe is 609.6 cm and the cross-sectional area is 20.27 cm².

$$R = \frac{300 \ \Omega \cdot cm \times 609.6 \ cm}{20.27 \ cm^2} = 9022 \ \Omega$$

The resistance of electrical wires can be found in various reference manuals. Electrical wire sizes in the United States are usually given in **American Wire Gage (AWG)** with cross-sectional area given in something called circular mils (cmil) which is simply the diameter of the wire in thousandths of an inch squared. When these dimensions are used the resistivity is generally designated with the letter K. For this example determine the resistance of a 200 ft length of size 12 AWG copper wire which has a cross-sectional area of 6530 cmil and a resistivity of 11.8 Ω -cmil/ft. Values of electrical resistivity of typical metal conductors in these units can be found in various reference manuals. The total resistance of this length of copper wire is 0.361 Ohm.

R = $\frac{11.8 \,\Omega \cdot \text{cmil/ft} \times 200 \,\text{ft}}{6530 \,\text{cmil}}$ = 0.361 Ω

Electrical resistance of a material changes significantly with a change in temperature. For most applications the change in resistance for a given change in temperature is taken as a constant, but it is not a constant. In the few cases where extreme accuracy is important, care must be taken in determining resistance of a conductor at a specific temperature. For metal conductors, the resistance increases as the temperature increases and decreases as the temperature decreases.

The change of resistance of electrical conductors with a change in temperature cannot be ignored. The change in resistance over typical operating conditions of a conductor can often change the current carrying capacity of the wire enough to cause problems. The current carrying capacity of electrical wires in the electrical codes is generally based upon a specific wire operating temperature. If that temperature is exceeded a dangerous condition may result. A general rule of thumb can be used to estimate the change in resistance for a given change in the wire temperature. For copper and aluminum wire at typical operating temperatures, the resistance change is about 8% for every 25°C change in operating temperature of the wire. If the resistance of a given length and size of copper wire is 1.0 ohm at 50°C it will have a resistance of 1.08 ohm at 75°C and 0.92 ohm at 25°C. A more accurate method of determining the resistance of a wire is expressed by Equation 211.3. The temperature coefficient of resistivity (α) for copper is approximately $\alpha_{cu} = 0.00323$, and for aluminum the coefficient is approximately $\alpha_{al} = 0.00300$. These values also change as the temperature changes.

$$R_2 = R_1 [1 + \alpha (T_2 - T_1)]$$
 Eq. 211.3

Conductivity is the measure of the ability of a material to conduct electrical charge. It is the opposite of resistivity. Conductivity is the reciprocal of resistivity (ρ) as shown by Equation 211.4. The unit of measure of conductivity is the Siemen per meter (Sometimes more conveniently Siemens per cm). The **Siemen** is the SI metric unit of conductance which is simply the reciprocal of resistance in ohms. The term Siemen replaces the old term mho for conductance. The Siemen and the mho are equivalent, Metals have a high value of conductivity. The symbol for the **unit Siemen is S.** The symbol for conductivity is the lower case Greek letter Sigma σ .

$$\sigma = \frac{1}{\rho}$$
 Eq. 211.4
 σ is electrical conductivity in S/cm

 ρ is electrical resistivity in Ω -cm

Some conductors exhibit a unique characteristic called *superconductivity* when cooled to an extremely low temperature. *Superconductivity is a condition where the resistance of a material drops to zero*. Superconductivity permits extremely high levels of current to flow through an extremely small conductor while producing little or no heat. Most conductors that exhibit this property must be cooled to less than 4°K in order to become superconducting. Liquid helium is used to cool the conductors to this extremely low temperature. Some materials will become superconducting at near 80°K and these materials can be cooled using liquid nitrogen. A temperature of 0°K is called **absolute zero** which is the lowest temperature possible. *Absolute zero is about minus 273°C or minus 460°*F.

The product of voltage and current is **power**. Power is the rate of expending energy. Voltage is Joules per Coulomb and current is Coulombs per second. The product of volts and current is Joules per second. The unit of measure of electrical power is the Watt where one Watt is equal to one Joule per second. For comparison purposes, **one horsepower is equal to 746 watts.** Electricians need to memorize the watts to horsepower conversion.

1 Watt = 1 Volt × 1 Ampere = Joules/Coulomb × Coulombs/second = Joules/second

1 horsepower = 746 watts

Electron Flow in a Conductor: When an electrical wire is connected to a source of voltage the electrons within the wire are exposed to an electric field which forces them to move through the wire in the direction towards the positive charge of the electric field. *The free electrons all along the wire are simultaneously exposed to the electrical field and all of them simultaneously move in the direction of the field.* This instant movement of electrons can give the impression that the individual electrons are moving very fast through the wire, but in fact any one electron actually is moving quite slowly along the wire. If the level of current in amperes flowing through a wire is known and the cross-sectional area of the wire is known in square meters, the actual speed of an electron moving through the wire can be estimated. *Equation 211.5 can be used to estimate the velocity of electron flow through a metal conductor* where the current flow is due to a steady voltage in one direction.

Example: As an example assume **one ampere** of direct current is flowing through a size 14 AWG copper wire with a cross-sectional area of $2.08 \times 10^{-9} \text{ m}^2$. To make the calculation it must be recognized that an ampere is one Coulomb per second, the charge of a single electron is 1.602×10^{-19} Coulomb, and there are an estimated 10^{29} free electrons in one cubic meter of a metal such as copper. As it turns out the speed of a single electron flowing through this size 14 AWG copper wire with one ampere flowing is only 1.8 meters per minute which is <u>5.91</u> ft/min. With one ampere of direct current flowing through a size 14 AWG wire it takes approximately 10 seconds for an electron to travel one foot along the wire. With alternating current the electrons move first in one direction then reverse and move in the other direction.

$$v = 0.625 \times 10^{-10} \frac{I}{A}$$

Eq. 211.5

Where *I* is in amperes and A is the cross-sectional area of the wire in m^2 . Velocity (v) is in meters per second. The constant is the reciprocal of the estimated amount of charge in a cubic meter of metal such as copper (m^3/C).

Current will only flow when a circuit is completed: Electrical current will have a source where energy is applied to the charges. *Current will leave a source and return to that same source.* If the circuit is interrupted, current flow will stop. Current does not originate from one point and go to another. Current always returns to the original source. In a direct current circuit a battery may be the source. Current originates at the battery and returns to the battery terminal of opposite polarity as illustrated in *Figure 211.4.* This is an extremely important concept to keep in mind when troubleshooting problems in electrical circuits.

With alternating current (ac) circuits the transformers are common to change the voltage from one level to another. Energy is transferred from one transformer winding to another by means of a magnetic field. The input circuit and the output circuit of the transformer are different circuits unless they are deliberately connected to each other. The winding of a transformer that supplies a circuit is often considered to be the source for that circuit and all current that originates from that transformer winding will return to that same transformer winding. The current in an alternating current circuit first flows in one direction then reverses and then flows in the opposite direction. Keep in mind that at any one instant in time the current in an ac circuit can be examined as though it is a direct current circuit. A circuit with a transformer winding acting as the source is shown in *Figure 211.5*.



Figure 211.4 A battery is the source of electrical energy in this circuit, and the charge that originates at one battery terminal returns to the battery terminal of opposite polarity.



Figure 211.5 The secondary winding of a transformer is often the source in an alternating current circuit. At any *instant* the current flows in one direction the same as for a direct current circuit.

Magnetism & Electrical Current: Electrical current is the flow of electrical charge, positive charge in one direction and an equal quantity of negative charge in the opposite direction. When current flows through a wire a magnetic field builds up around the wire. (*Figure 211,6*) The magnetic field all along the wire encircles the wire in a particular direction. Notice when a person views the wire cross section with the current flowing towards the person, the magnetic flux encircles the wire in a counter-clockwise direction. When the current is flowing away from the observer as seen in the (left hand diagram) the magnetic flux encircles the wire cross section. More about magnetic fields in *Tech Note 217*.



Figure 211.6: Viewing the wire cross section with current flowing towards the observer (right diagram) the magnetic field encircles the wire in the counter-clockwise direction.