

Parameters Influencing Magnetic Flux Density

Electrical current completes a circuit from one terminal of a source and back to the other terminal of the source. The current will seek a path that has the least resistance. Current will flow in all paths, but there will be more current flowing in paths with little resistance. A magnetic flux will complete a circuit from a bar magnet or an electromagnet from the North pole to the South pole. Some materials such as iron and steel are good conductors for a magnetic flux and other materials such as air are poor conductors for a magnetic flux. **Reluctance** is the quantity that opposes magnetic flux. A material that is a good conductor for a magnetic flux has a low reluctance. The unit of reluctance is amperes per weber which can be considered the current required to obtain a weber of magnetic flux. The opposite of reluctance is **permeability**. A material with a high permeability is a good path for magnetic flux.

Magnetic Circuit: When creating a strong *magnetic flux density (B)* with an electromagnet the goal is to achieve as high a flux density as possible with the lowest magnetic field intensity. *Magnetic field intensity (H)* is the number of amperes per unit length of a magnetic circuit such as amperes per inch or amperes per meter. A path through the center of the electromagnet and completing a loop around the outside is needed with as high a *permeability (μ)* as possible in order to maintain a high magnetic flux density. If a path of high permeability is provided the magnetic flux will be concentrated in the magnetic material and not spread out into surrounding space. The permeability of some common magnetic path materials is shown in *Figure 317.1*. The materials with high permeability have a drawback called **saturation**. A point is reached where the magnetic flux density does not increase linearly when magnetic field intensity is increased.

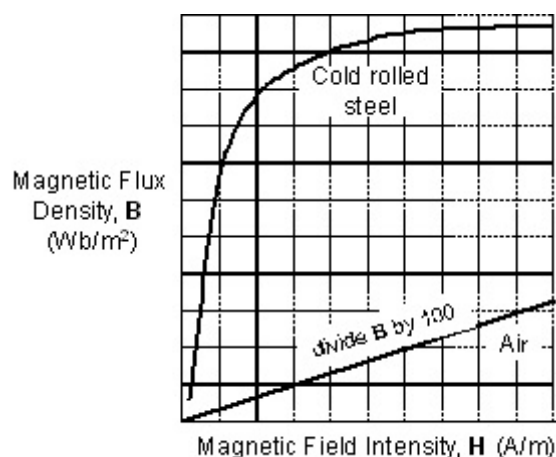


Figure 317.1 Magnetic flux density (B) increases in a material as the magnetic field intensity (H) is increased, but the materials with the best magnetic properties reach a point called **saturation** where an increase in magnetic field intensity results in only a minimal increase in magnetic flux density.

Air and a vacuum as a path for a magnetic flux do not saturate but they have very low permeability. The magnetic flux spreads out and is not concentrated. A high flux density is hard to achieve in air. Similar to insulation around an electrical conductor that prevents loss of current from a conductor most of the magnetic flux will remain in a steel path with little flux **leakage** into the surrounding air. The flux is then concentrated in the steel and a high flux density can be achieved. The ratio of permeability of magnetic path materials to air is only a few thousand to one so there will be some small flux leakage to the surrounding air.

Although copper and aluminum are excellent electrical conductors, they make very poor conductors for a magnetic flux. Materials made with iron make the best conducting medium for a magnetic flux such as cold rolled steel.

Magnetic Permeability: The symbol for *magnetic permeability* is the Greek letter **μ** and the units are *Henry per meter (H/m)*. The *Henry* is the unit of measure of inductance and is equal to *one volt-second per ampere (V·s/A)*. The **permeability of free space** or a vacuum (μ_s) is equal to **$1.256 \times 10^{-6} \text{ H/m}$** . It is customary to describe permeability of materials relative to the permeability of free space. Approximate values of **relative permeability** (μ_r) are 1.0 for air, 1,000 for cast steel, 4,000 for sheet steel, and 5,000 for cast iron. The **permeability of a magnetic path** is determined by multiplying free space permeability by the relative permeability ($\mu_r \times \mu_s$).

Magnetic Flux: It was discovered when fine particles of iron were placed near a magnet the particles would stick together and line up between the ends of the magnet in strings or lines. This is illustrated in *Figure 317.2*. Even though it was later discovered that actual lines do not exist it was convenient to refer to a magnetic flux as “lines of force”. **Magnetic flux (ϕ)** can be quantified and today the common unit of magnetic flux is the *Weber (Wb)*. Another unit of magnetic flux is the Maxwell and it takes 10^8 Maxwell to equal one Weber. Compared to the concept of lines of force 1 Weber is equal to 100 million lines (10^8). Magnetic flux has direction and it completes a magnetic circuit. *The direction of a magnetic flux outside of the magnet is considered as pointing from the North pole to the South pole of the magnet as shown in Figure 317.2*. A Weber is also equal to one volt-second ($\text{Wb} = \text{V} \cdot \text{s}$). Webers and Henrys will sometimes both be found in the same equation. *One Henry is one Weber per ampere ($H = \text{Wb/A}$), or one Weber is equal to one Henry-ampere ($\text{Wb} = H \cdot A$)*. Be careful with the use of the letter “H” in the following analysis. The unit Henry and variable magnetic field intensity are both represented by the letter “H”.

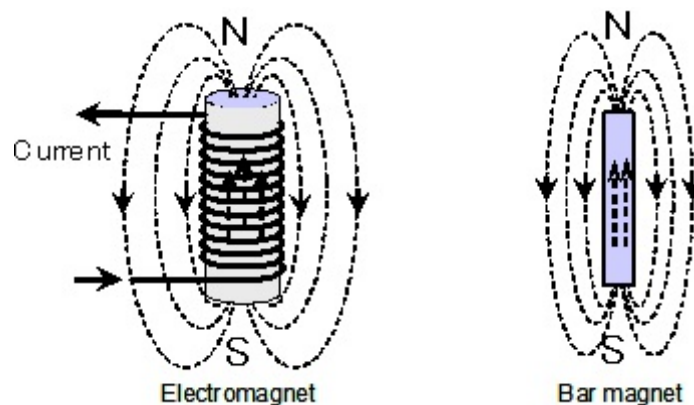


Figure 317.2 Whether a permanent magnet or an electromagnet, a flux resembling lines develop, completing a circuit from North to South outside of the magnet and from South to North inside the magnet.

Magnetic flux does not require some physical material to complete a circuit. It can act through free space (vacuum). The behavior of a magnetic flux in air and many nonmetallic materials acts essentially the same as a magnetic flux in free space. Air is a poor medium for a magnetic flux. The *flux spreads out when it must act through air or free space* and even though a strong magnet may be used, at any point away from the magnet the strength of the flux may be weak. It was discovered that *iron and steel provide an excellent path for a magnetic flux*.

Terminology and Units: An analogy can be drawn between current flow in a circuit and flux in a magnetic circuit. Ohm's law states that current flow in a circuit is equal to the voltage divided by the resistance. For a magnetic circuit *magnetic flux* (ϕ) is analogous to current, a quantity called magnetomotive force or mmf (F) is analogous to voltage, and **reluctance** of the magnetic circuit (R) is analogous to resistance of an electrical circuit. The magnetic circuit equation analogous to Ohm's law is that the amount of flux (ϕ) is equal to the magnetomotive force (F) divided by the reluctance (R) of the magnetic circuit. In the real world the calculations can get quite complex. The following discussion is intended to gain a basic understanding of magnetic principles and assumptions have been made to simplify the concepts and calculations. In many cases, however, these simplifications are valid and can be made to obtain approximate solutions to real problems.

Electrical **current flow (I)** is measured in *amperes* (1 Ampere = 1 Coulomb/second) and a magnetic flux is produced around a conductor through which current is flowing. The *magnetic flux intensity is proportional to the current flow*.

I = current flow in the coil (A)

The unit of measure of *length for a magnetic flux circuit is in meters*. The cross-sectional area of the magnetic flux path perpendicular to the direction of the flux is in square meters. For the purpose of the following examples the mean length of the magnetic flux circuit will be used and it will be assumed the flux is evenly distributed across the cross-section of the flux path.

L = length of the magnetic circuit (m)

A = cross-sectional area of the magnetic circuit (m²)

A strong magnetic flux can be created by passing current through wire tightly wound into a coil as shown in *Figure 317.2*. The *magnetic flux of each wire will add* to form a strong magnetic flux. An important parameter for an electromagnet is the **number of turns of wire (N)** forming the coil.

N = number of turns of the coil (no units)

The excitation required to produce a magnetic flux by passing current through a coil is called the **magnetomotive force (F_{mmf})** which is in *units of total amperes* flowing in a circle around a coil. The units ampere-turns is often used but turns is not really a unit. Turns are necessary to increase the amperes flowing in a circular path. *Magnetomotive force (mmf)* is the total current of the coil which is an accumulation of the current in each turn or winding as indicated by *Equation 317.1*. Tightly winding this coil to a core material will result in a **high level of linkage** of this magnetomotive force to the steel core material. This magnetomotive force is exerted over the total length of the magnetic core circuit.

F = N × I magnetomotive force (A or Ampere-Turns) **Eq. 317.1**

A magnetic circuit extends from one pole of a magnet through some medium and back to the other pole, and then through the magnet to the original pole. Magnetic flux like electrical current completes a circuit back to the source. When air or free space is the medium surrounding a magnet the length and cross-sectional area of the flux path is limitless. Experimental results can be used to describe the distribution of the flux in air about a magnet. When a material such as iron or steel is used as the path for the magnetic flux most of the flux will be confined within the material and the magnetomotive force is considered to be acting over the total length of the magnetic circuit. *The intensity of a magnetic field will be greater when the magnetic circuit is shorter.* Every material offers at least some resistance (called reluctance for magnetic materials) to a magnetic flux and the shorter the path the higher the resulting intensity. It is important to know the *magnetic field intensity* (H_{mf}) which is the magnetomotive force, (F_{mmf}) divided by the length (L_{core}) of the magnetic circuit. Since *magnetomotive force is in amperes and magnetic circuit length is in meters magnetic field intensity is in amperes per meter* as indicated by Equation 317.2.

$$H = \frac{F}{L} \quad \text{magnetic field intensity} \quad (\text{A/m}) \quad \text{Eq. 317.2}$$

Magnetic permeability is a measure of how good a material is as a path for a magnetic flux. The higher the permeability the better the material is as a path for a magnetic flux. Generally *permeability is expressed as a number relative to the magnetic permeability of free space.* To determine the actual *permeability* of a material (μ), the *relative permeability* of the material (μ_r) is multiplied by the *magnetic permeability of free space* (μ_s) as described by Equation 317.3. The *unit of permeability is the Henry per meter (H/m)*. One Henry of inductance is equal to one volt being produced by a time rate of change of current of one ampere per second. The units of a Henry are volt seconds per ampere.

$$\mu = \mu_r \times \mu_s \quad \begin{array}{ll} \text{magnetic permeability} & (\text{H/m}) \text{ also } (\text{Wb/A}\cdot\text{m}) \\ \mu_r \text{ relative permeability} & (\text{no units}) \\ \mu_s \text{ permeability of free space} & (1.256 \times 10^{-6} \text{ H/m}) \end{array} \quad \text{Eq. 317.3}$$

Frequently an objective is to determine the *magnetic flux density* at a point in a magnetic circuit or to determine the current level and number of turns needed at a coil to produce a particular magnetic flux density at a point in a magnetic circuit. The larger the permeability of the magnetic circuit the smaller the current required to produce the flux density required. The unit of *magnetic flux density in Weber per square meter (Wb/m^2)*. Magnetic flux density can be determined by multiplying the core permeability (μ) by the magnetic field intensity (H_{mf}) as shown in Equation 317.4.

$$B = \mu \times H \quad \text{Magnetic flux density} \quad (\text{Wb/m}^2) \quad \text{Eq. 317.4}$$

The total magnetic flux is equal to the magnetic flux density (B) times the cross-sectional area of the magnetic circuit which is usually the cross-sectional area of the magnetic core, shown by Equation 317.5. Equation 3127.6 is similar to Ohm's law for electrical circuits. The total magnetic flux (ϕ) is equal to the magnetomotive force (F) divided by the total reluctance of the magnetic circuit (R) as shown in Equation 317.6. This is a useful way of analyzing magnetic circuits that consist of a *magnetic core and one or more air gaps*. For some magnetic circuits, there may be a portion of the circuit that forms a parallel path such as shown in Figure 317.3. The sum of the magnetic flux in each parallel path will add up to the total magnetic flux. It is common to have two identical parallel paths completing a magnetic circuit, in which the magnetic flux will divide equally with half of the flux in each parallel path.

$$\phi = B \times A \quad \text{Total magnetic flux} \quad (\text{Wb}) \quad \text{Eq. 317.5}$$

$$\phi = \frac{F}{R} \quad (\text{Wb}) \quad \text{Eq. 317.6}$$

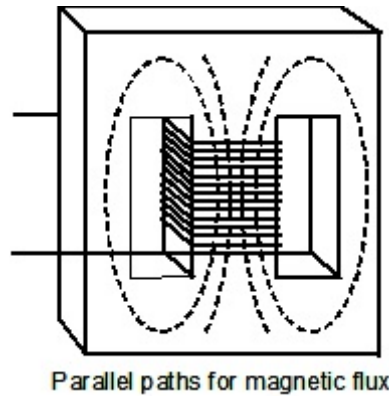


Figure 317.3 Some magnetic cores are constructed with a parallel path and the flux divides to take both paths available.

Reluctance: Reluctance is the resistance to magnetic flux within a magnetic circuit. Like resistance in an electrical circuit the total reluctance of a magnetic circuit with different dimensions or with different core materials in series is equal to the sum of the reluctance of each section of the magnetic circuit. Assuming the magnetic flux is uniformly distributed across the magnetic circuit material the reluctance of the circuit is proportional to the length of the circuit and inversely proportional to the cross-sectional area of the material. The *unit of reluctance is the reciprocal of Henrys*. Reluctance is also amperes per weber. Be careful with the letter “A” which represents area in a formula and amperes in the dimensions. The total reluctance of a core represents the amperes required to achieve one weber of flux. Equation 317.7 is used to determine the reluctance of each section in series that makes up a magnetic core. The total reluctance is the sum of each section’s reluctance. The following example shows how the magnetic flux density of a core with uniform dimensions is determined.

$$R = \frac{L}{\mu \times A} \quad \text{reluctance} \quad (1/H) \text{ also } (A/Wb) \quad \text{Eq. 317.7}$$

Example 1: The following example will illustrate how magnetic flux density (B) can be determined for an electromagnet coil around a steel core that makes a complete magnetic circuit as illustrated in Figure 317.4. The mean length of each side of the steel core is 12 cm. The cross-section of the steel core is 4 cm on each side. There are 400 turns of wire around one leg of the core with a current of 0.2 amperes. The relative permeability of the core material (μ_r) is 4,000. The following assumptions can be made in order to obtain an approximate result without difficult calculations. It will be assumed that all of the magnetic flux produced by the coil will be placed into the core (flux and coil are linked). There will be no core leakage (all flux remains within the core) and the density of the flux across the core cross-section is uniform.

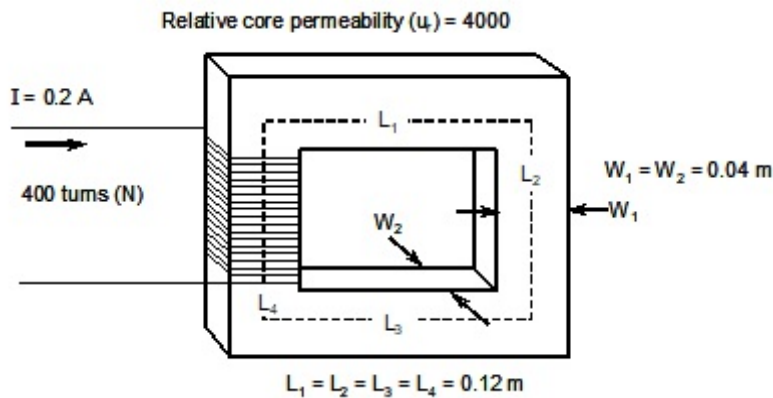


Figure 317.4 Determine the magnetic flux density (B) for a steel core with relative permeability (μ_r) of 4,000 and 0.2 amperes flowing in a 400 turn coil.

Solution: First determine the magnetomotive force (F)

$$F = N \times I = 400 \text{ turns} \times 0.2 \text{ A} = 80 \text{ A-turns}$$

Next determine magnetic field intensity (H). The mean magnetic circuit length is 0.48 m.

$$L = L_1 + L_2 + L_3 + L_4 = 4 \times 0.12 \text{ m} = 0.48 \text{ m}$$

$$H = \frac{F}{L} = \frac{80 \text{ A}}{0.48 \text{ m}} = 167 \text{ A/m}$$

Finally determine the magnetic flux density (B) but first determine the permeability of the magnetic circuit.

$$\mu = \mu_r \times \mu_s = 4,000 \times 1.256 \times 10^{-6} \text{ H/m} = 5.029 \times 10^{-3} \text{ H/m (Wb/A}\cdot\text{m)}$$

$$B = \mu \times H = 5.029 \times 10^{-3} \text{ Wb/A}\cdot\text{m} \times 167 \text{ A/m} = 0.839 \text{ Wb/m}^2$$

Air Gap in a Magnetic Circuit: Many magnetic circuits have a core air gap. An example is an electric motor where a magnetic circuit must be completed through a rotor. A solenoid is another example of a magnetic circuit with an air gap. The permeability of air is very low compared to iron or steel and the air gap represents a significant reluctance in the magnetic circuit even though the length of the air gap is very small. Since air surrounds the magnetic core as well as in the gap, the magnetic flux tends to expand outward at the gap as shown in Figure 317.5. This expanding of the magnetic flux at the air gap is called **fringing**. The effective cross-sectional area of the air gap as a magnetic circuit is larger than the cross-sectional area of the magnetic core as indicated by Equation 317.8. The flux density across the air gap is not uniform but some assumptions can be made for the purpose of analysis that have been found to give a reasonable result. The flux density is assumed to be uniformly distributed across a cross-section that is equal to the products of the core dimensions (w_1 & w_2) increased by the length of the air gap (L_g).

$$\text{Air gap cross-section} = (w_1 + L_g) \times (w_2 + L_g)$$

Eq. 317.8

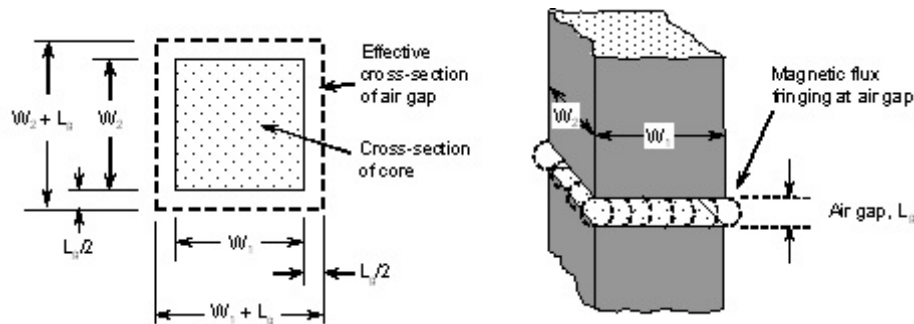


Figure 317.5 Fringing of the magnetic flux at an air gap will occur and for analysis purposes, the cross-sectional area of the flux through the air gap can be assumed to be the product of the dimensions of the core increased by the length of the air gap (L_g).

Example 2: The following example will illustrate how magnetic flux density (B) can be determined for an electromagnet coil around a steel core that makes a magnetic circuit with an air gap of 0.002m as illustrated in *Figure 317.6*. The mean length of each side of the rectangular magnetic circuit is 12.0 cm. The steel core on the side of the magnetic circuit with the air gap is reduced to a length of 0.059 m. The cross-section of the steel core is 4.0 cm on each side. There are 400 turns of wire around one leg of the core with a current of 0.2 amperes. The relative permeability of the core material (μ_r) is 4,000. The following assumptions can be made in order to obtain an approximate result without difficult calculations. It will be assumed that all of the magnetic flux produced by the coil will be placed into the core (flux and coil are linked). There will be no core leakage (all flux remains within the core except at the air gap) and the density of the flux across the core and air gap cross-section is uniform.

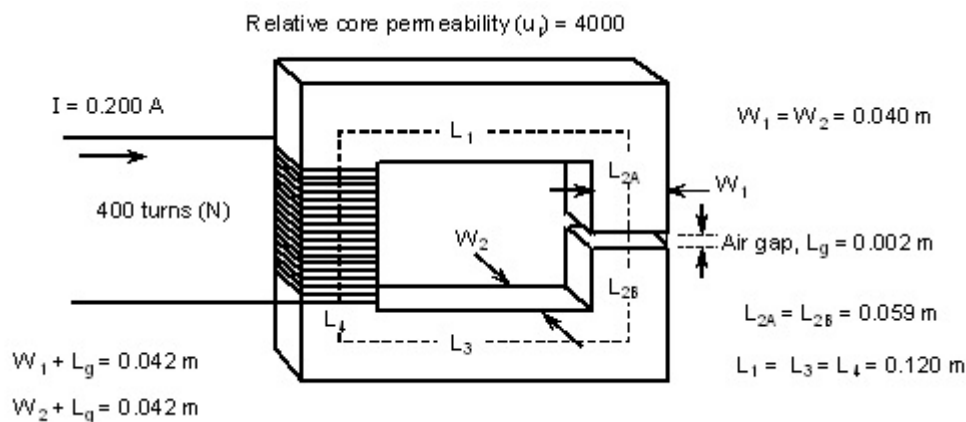


Figure 317.6 A magnetic circuit consists of a rectangular steel core with 400 turns and a current flow of 0.2 amperes. One leg of the steel core has an **air gap** with a length of 0.002m.

Solution: To determining the flux density (B) with a magnetic circuit consisting of sections in series not having the same permeability, using *reluctance* rather than permeability is a more efficient method of analysis. *Reluctance* can be determined for the steel core and the air gap with the total magnetic circuit reluctance as the sum of the reluctance of each series section.

Solution: First determine the magnetomotive force (**F**)

$$F = N \times I = 400 \text{ turns} \times 0.2 \text{ A} = 80 \text{ A-turns}$$

Next determine dimensions of the sections of the magnetic circuit, magnetic permeability of each section, and then the reluctance of each section of the magnetic circuit.

Steel core: $L_C = L_1 + L_{2A} + L_{2B} + L_3 + L_4$
 $= 0.120\text{m} + 0.059\text{m} + 0.059\text{m} + 0.120\text{m} + 0.120\text{m} = 0.479\text{m}$

$$A_C = w_1 \times w_2 = 0.040\text{m} \times 0.040\text{m} = 1.60 \times 10^{-3} \text{ m}^2$$

$$\mu_C = \mu_r \times \mu_s = 4000 \times 1.256 \times 10^{-6} \text{ H/m} = 5.029 \times 10^{-3} \text{ H/m}$$

$$R_C = \frac{L_C}{\mu_C \times A_C} = \frac{0.479\text{m}}{5.029 \times 10^{-3} \text{ H/m} \times 1.60 \times 10^{-3} \text{ m}^2} = 0.059 \times 10^6 \text{ 1/H}$$

Air gap: $L_g = 0.002\text{m}$

$$A_g = (w_1 + L_g) \times (w_2 + L_g) = (0.040\text{m} + 0.002\text{m}) \times (0.040\text{m} + 0.002\text{m})$$

$$= 1.76 \times 10^{-3} \text{ m}^2$$

$$\mu_g = \mu_r \times \mu_s = 1 \times 1.256 \times 10^{-6} \text{ H/m} = 1.256 \times 10^{-6} \text{ H/m}$$

$$R_g = \frac{L_g}{\mu_g \times A_g} = \frac{2 \times 10^{-3} \text{ m}}{1.256 \times 10^{-6} \text{ H/m} \times 1.76 \times 10^{-3} \text{ m}^2} = 0.905 \times 10^6 \text{ 1/H}$$

Total reluctance: $R_T = R_C + R_g = 0.059 \times 10^6 \text{ 1/H} + 0.905 \times 10^6 \text{ 1/H}$
 $= 0.964 \times 10^6 \text{ 1/H also (A/Wb)}$

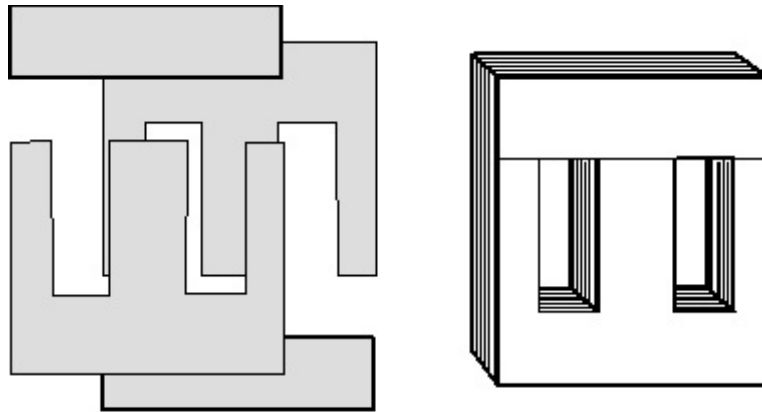
Total Flux: Next determine the total flux (**φ**) for the magnetic circuit.

$$\phi = \frac{F}{R_T} = \frac{80 \text{ A}}{0.964 \times 10^6 \text{ A/Wb}} = 82.97 \times 10^{-6} \text{ Wb}$$

Flux Density: Finally determine the magnetic flux density (**B**) of the magnetic circuit with the air gap. The following flux density is determined for the steel core. If a value was determined for the air gap it would only be an approximate average value. The flux density across the cross-section of the air gap is not uniform. Compared with the previous example where there was no air gap, the flux density of the same core was 0.839 Wb/m². Even a small air gap causes a major reduction in the flux density of a magnetic circuit.

$$B = \frac{\phi}{A_C} = \frac{82.97 \times 10^{-6} \text{ Wb}}{1.60 \times 10^{-3} \text{ m}^2} = 0.052 \text{ Wb/m}^2$$

Magnetic Core Construction: When an electromagnetic coil is energized with alternating current the magnetic flux is constantly in motion and cutting across the magnetic core. Since the magnetic core is usually steel or iron and an electrical conductor, a current will be induced to flow in the core. This *core current flow results in loss of energy and undesirable heat produced*. To minimize current flow in the magnetic core (*eddy currents*), the core of many devices such as solenoids, transformers, and motors consist of thin sheets of steel with a very thin electrical insulating coating similar to the diagram in *Figure 317.7*. Unfortunately the insulation has a low permeability and creates a thin gap between the sheets of steel which causes a slight decrease in the overall permeability of the core but it is necessary to prevent excessive heat and losses.



Core is constructed of thin insulated sheets of steel to minimize eddy currents

Figure 317.7 A magnetic core is generally constructed of thin insulated sheets of steel to minimize **eddy currents** in the core.

Table 317.1 Common units used in magnetic circuits and equivalent units.

Quantity	Symbol	Name	Units	Units	Units	Units
Inductance	L	Henry, H	H	Wb/A	V•s/A	J•s ² /C ²
Magnetic flux	ϕ	Weber, Wb	Wb	V•s	H•A	J•s/C
Magnetic field intensity	H		A/m		C/m•s	
Magnetic flux density	B	Tesla, T	Wb/m ²	H•A/m ²	J•s/C•m ²	
Magnetic permeability	μ		H/m	Wb/A•m	V•s/A•m	J•s ² /C ² •m
Magnetomotive force	F		A		C/s	
($F=\phi\cdot R$)						
Reluctance ($R=L/\mu\cdot A$)	R		1/H	A/Wb	C ² /J•s ²	
Force	f	Newton, Nt			J/m	
Mass	m	kilogram, kg			J•s ² /m ²	
Voltage	E	Volt, V			J/C	
Current	I	Ampere, A	A		C/s	
Charge	q	Coulomb, C	C			
Energy		Joule, J	J			
Time	t	Seconds, s	s			
Area	A		m ²			
Length	L		m			
Magnetic flux (Wb)					10 ⁸ lines	