Electrical Tech Note — 215

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Solving Circuit Problems

Typical electrical circuits consist of elements that are connected in series with each other, parallel with each other, and a combination of series and parallel. To solve these problems it is important to know how to use Ohm's law, and the power formula as well as the basic principles of a series and parallel circuit. The procedures discussed in this Tech Note are limited to circuits with only one voltage supply. For the purpose of this discussion, voltage will be represented by the symbol **E**, current by the symbol **I**, and resistance by the symbol **R**. Methods of analyzing more complex circuits are discussed in *Tech Note 230.*

Series Circuit Rules: A series circuit is one where there is only one path through all of the elements. An example of a series circuit with three components is shown in *Figure 215.1.* Here are the rules for a series circuit.

- The total current (I_T) for a series circuit is the same at all points in the circuit.

  $$I_T = I_1 = I_2 = I_3$$

- The total voltage (E_T) of a series circuit is the sum of the voltages across each component connected in series.

  $$E_T = E_1 + E_2 + E_3 \qquad \textbf{Eq. 215.1}$$

- The total resistance (R_T) of the series circuit is the sum of the resistance of each element connected in series.

  $$R_T = R_1 + R_2 + R_3 \qquad \textbf{Eq. 515.2}$$

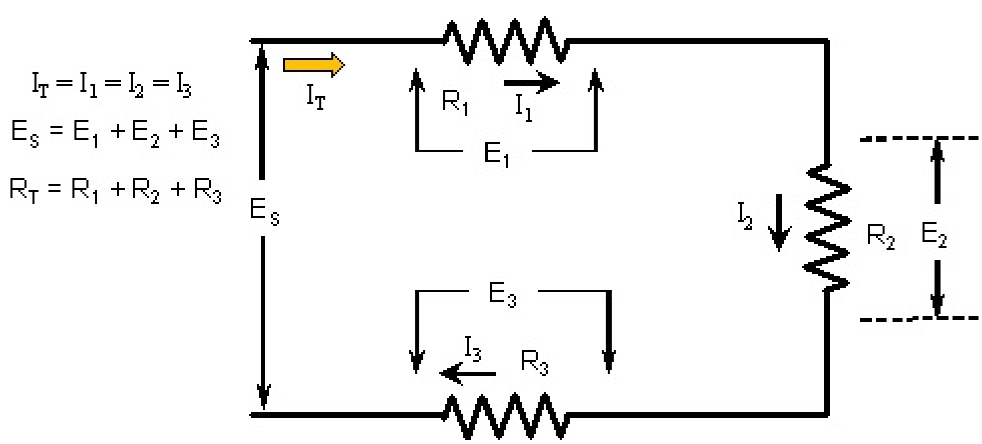

Figure 215.1 *The components are arranges so there is only one path through the circuit with the current the same at every point in the circuit.*

Parallel Circuit Rules: A parallel circuit is one where each component receives the same voltage and the total circuit current is the sum of the current through each component. The current will divide up and take each path available. The paths with the lowest resistance will have the highest current. An example of a parallel circuit is shown in *Figure 215.2.* Here are the rules for a parallel circuit.

- The voltage is the same across each component of a parallel circuit.

  $$E_T = E_1 = E_2 = E_3$$

- The total current flow (I_T) to components connected in parallel is the sum of the current through each component.

  $$I_T = I_1 + I_2 + I_3 \qquad \textbf{Eq. 215.3}$$

- The total resistance (R_T) of a group of components connected in parallel is always less than the value of the component with the smallest resistance.

- The total resistance (R_T) of a group of components connected in parallel can be calculated in several ways depending upon how many resistors are connected in parallel. The generic calculation for total resistance of parallel components is that the reciprocal of the total resistance is equal to the sum of the reciprocal of each parallel resistor as follows:

  $$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad \textbf{Eq. 215.4}$$

  Determine the lowest common denominator for the fractions and determine the sum. Then invert the value to get the total resistance. If there are only two resistors in parallel, it sometimes is easier to determine the total resistance by a method called product over sum using *Equation 215.5.*

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} \qquad \textbf{Eq. 215.5}$$

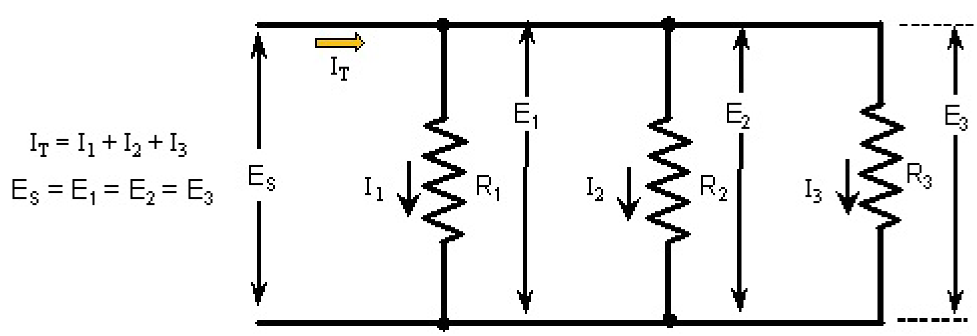

Figure 215.2 *The components are arranges so there are as many paths through the circuit as there are components in the circuit. The voltage is the same across each component and the total current is the sum of the current through each component.*

Solving Circuit Problems: It is important to be able to visualize the circuit and understand which components are connected in series and which are connected in parallel. One technique for visualizing a circuit is to arrange the circuit in a vertical pattern. Put the circuit voltage at the top of the diagram and draw the circuit vertically as shown in *Figure 215.3.* This is the same as the previous series circuit. Note there is only one path from top to bottom so the current will be the same at any point in the circuit. Now it is logical to see that the total voltage of the circuit will be the sum of the individual series voltages across the components. This technique will help to make complex circuits easier to understand.

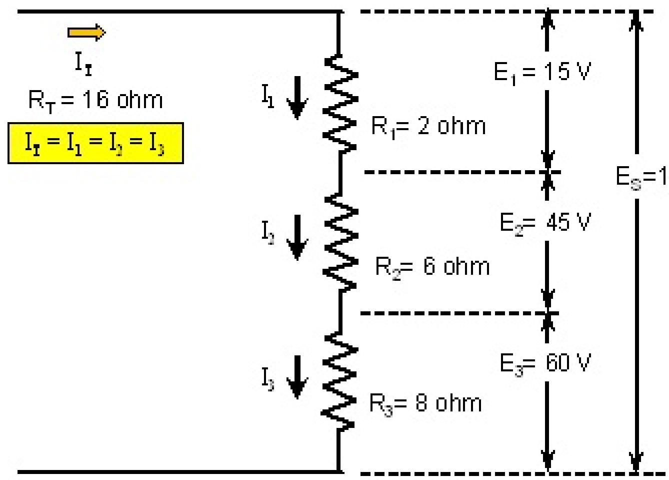

Figure 215.3 *This is a vertical diagram of the circuit of Figure 215.1 where one side of the voltage source is at the top and the other side is at the bottom of the diagram.*

Component Value Box: Sometimes it is easier to determine unknown component values of voltage, resistance, current, and power by placing the known values in a table. Using the component value box, it is easier to see when there are enough values to apply either Ohm's law or the power formula. *Table 215.1* is a component value box for the circuit of *Figure 215.4.* On the right hand side of *Figure 215.4* the circuit is redrawn vertically so it is easier to visualize.

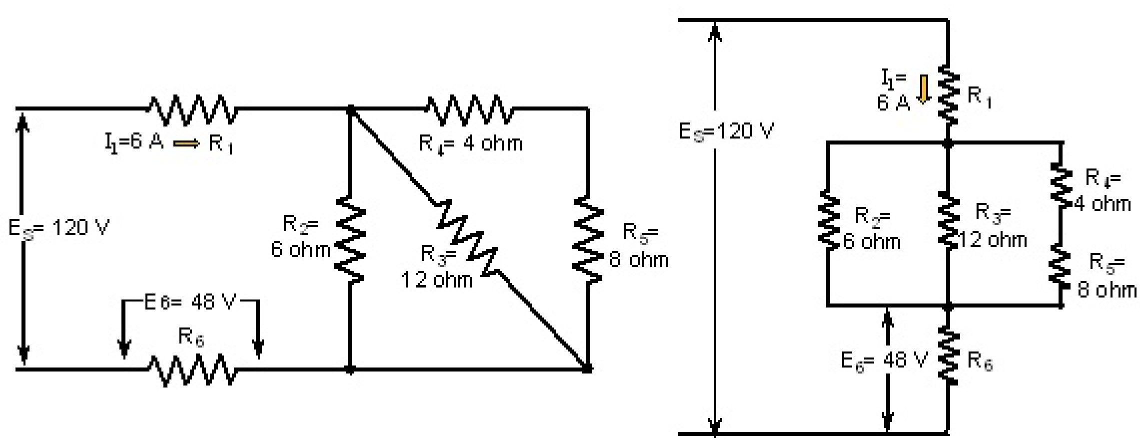

Figure 215.4 *A circuit consists of several components some of which are arranged in series and some are arranged in parallel. The diagram on the right is the same circuit redrawn vertically.*

Table 215.1 *This component value box can be used to record the known values to make it easier to determine when Ohm's law or the power equation can be applied.*

	Total	**1**	**2**	**3**	**4**	**5**	**6**
E	**120 V**	*54 V*(6)	*18 V*(4, 5)	*18 V*(4, 5)	*6 V*(9)	*12 V*(9)	**48 V**
I	*6 A*(1)	**6 A**	3 A(10)	*1.5 A*(10)	*1.5 A*(8)	*1.5 A*(8)	*6 A*(1)
R	*20 ohm*(2)	*9 ohm*(7)	**6 ohm**	**12 ohm**	**4 ohm**	**8 ohm**	*8 ohm*(2)
P							

1. The current of 6A through R_1 is the total current and will also pass through R_6.

2. Now the total circuit resistance and the resistance of R_6 can be determined using Ohm's law.

   R_T = 20 ohm R_6 = 8 ohm

3. R_4 and R_5 can be added to get a total resistance of that branch with is 12 ohm.

4. Note in the center of the circuit are three resistors in parallel. One has a value of 6 ohm, another has a value of 12 ohm, and the final branch has a value of 12 ohm. Apply any parallel resistor technique desired to find that these resistors have a combined value of 3 ohm. The circuit can be modified to look like *Figure 215.5.*

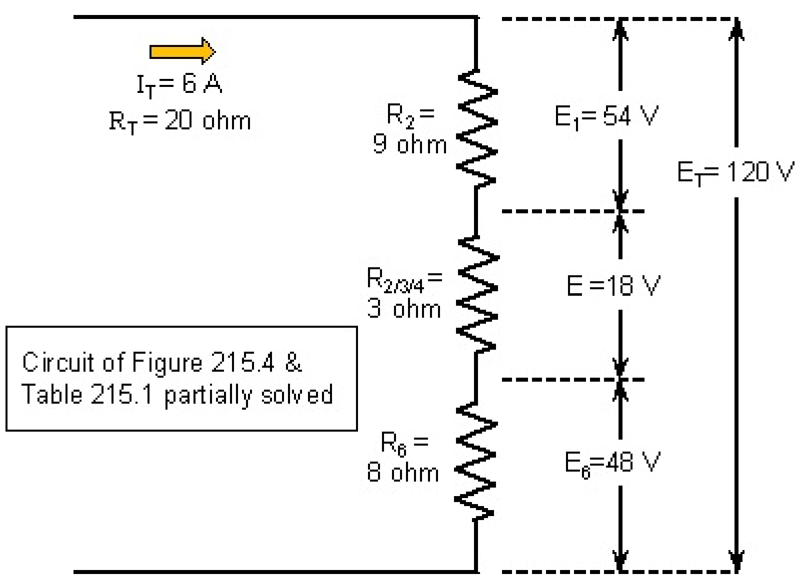

Figure 215.5 *Use the parallel resistor rule to find that a 3 ohm resistor in this circuit which replaces the three parallel resistors.*

5. It is known that 6 A will flow through the parallel section of the circuit, so multiply 6 A times 3 ohm (*Figure 215.5*) to determine the voltage across the parallel section is 18 V. The voltage across the 6 ohm and 12 ohm resistors will be 18 V.

6. To find the voltage across R_1 subtract 48 V and 18 V from the total 120V to get 54 V.

7. Now that the voltage and current are known for R_1, it is possible to calculate the value of R_1 by dividing 54 V by 6 A to get 9 ohm.

8. Now refer back to the right hand diagram of the circuit in *Figure 215.4.* Resistor R_4 and R_5 are in series and can be combined to be 12 ohm. From steps 4 and 5 above it was determined that there is 18 V across this 12 ohm. Using Ohm's law find there is 1.5 A flowing through this 12 ohm resistance.

9. Multiply the current and resistance for each resistor R_4 and R_5 to get the voltage across each resistor, E_4 = 6 V and E_5 = 12 V. These voltages add up to 18 V which is the total voltage across this branch.

10. Finally for R_2 and R_3 divide the voltage by the resistance to get the current, I_2 = 3 A, I_3 = 1.5 A. Add the current through each of the three parallel branches of *Figure 215.4* to get the total current of 6 A.

Power in Series and Parallel Circuits: If the power at each component of a circuit is known it does not matter whether the circuit components are in series, parallel, or any other configuration, the total power of the circuit is simply the sum of the power at each component. Here in *Figure 215.6* are three resistors connected in series. As shown in *Figure 215.1* and *Equation 215.1* the sum of the voltages across each resistor is equal to the supply voltage. For a series circuit there is only one current so the sum of the power at each resistor must total to the power of the entire circuit. Remember for a circuit consisting of resistors the power expended at each resistor is the product of the current and the voltage across that resistor. If the power is known for all elements in a circuit the total power of the circuit is the sum of the power for each element of the circuit *(Eq. 215.7).*

$$P = E \times I$$ **Eq. 215.6**

$$P_T = P_1 + P_2 + P_3 +$$ **Eq. 215.7**

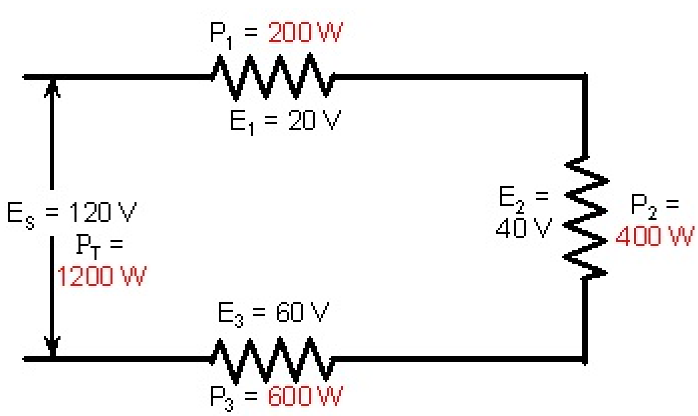

Figure 215.6. *For a series circuit the total power expended by the circuit is the sum of the power at each resistor of the circuit.*

In the case of a circuit where resistors are connected in parallel (*Figure 215.7*) each resistor is supplied by the circuit source voltage (E_S). The current flowing through each parallel resistor depends upon the value of resistance. The total current of the circuit is the sum of the current through each resistor. As shown in *Figure 215.2* and *Equation 215.3* the total current of the circuit of the parallel circuit is the sum of the current through each resistor. Likewise the total power of the parallel circuit is the sum of the power of each resistor (*Figure 215.7*).

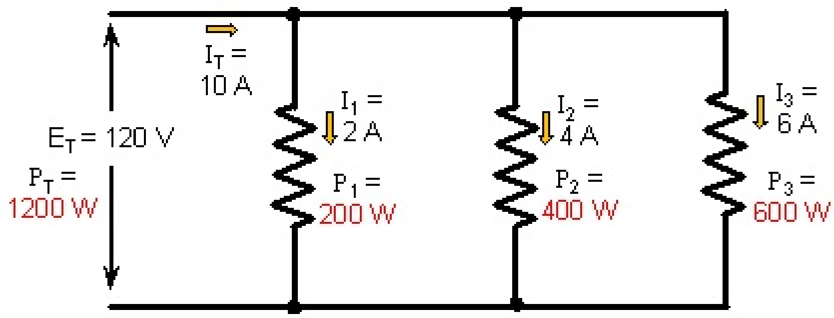

Figure 215.7. *When resistors are connected in parallel the power of each resistor will add to become the total power of the circuit.*

The circuit of *Figure 215.8* has both series and parallel resistors. It is not all series or all parallel. The total power of the circuit also is simply the sum of the power at each resistor as shown in the diagram. (P_{Total} = 180 W + 256 W + 64 W + 700 W = 1200 W)

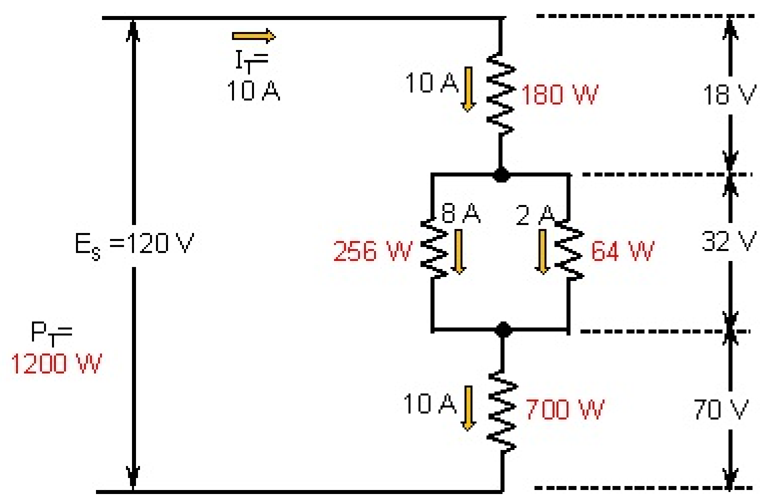

Figure 215.8. *For this circuit consisting of resistors that are connected in a combination of both series and parallel the total power of the circuit is simply the sum of the power of each resistor.*

Delta – T Transformation: Sometimes a circuit is encountered where the techniques of circuit reduction to simplify a circuit to a single resistor and voltage supply is not possible. Within the circuit three resistors can be found that form either a delta shape or a "T" shape. There is a transformation technique where a delta can be transformed into a "T" or a "T" shape can be transformed into a delta. This transformation technique makes analysis of the circuit much simpler. This transformation technique is discussed in detail in *Tech Note 215B*.