MAEC
Michigan Agricultural Energy Council

Electrical Tech Note — 223B

Agricultural Engineering Department
Michigan State University

Power in AC Circuits

Power is the rate of doing work or the rate of expending energy. The electrical unit of power is the watt. One watt is equal to one joule per second. In a dc circuit or an ac circuit where the load is a resistor the power expended by the circuit is the product of the voltage and current. *Voltage* is *joules per coulomb* and *current* in amperes is *coulombs per second*. The product of voltage and current yields joules per second. Power expended by a dc circuit can be determined by measuring the voltage and current separately and multiplying their values. As illustrated in *Figure 223.1* an ac circuit supplying only a resistive load (such as an incandescent lamp), the power in watts is equal to the product of the rms value of the voltage and current.

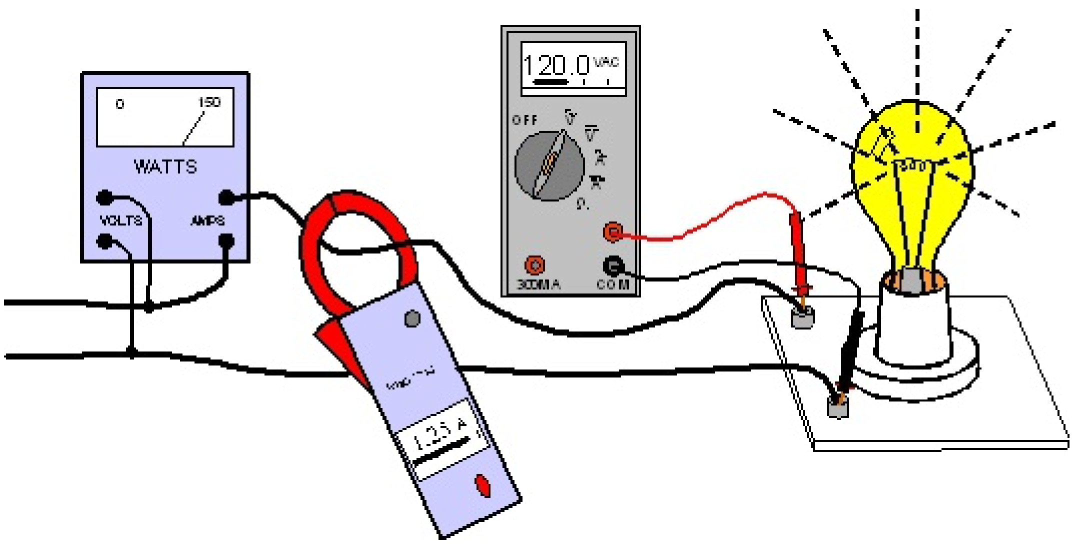

Figure 223.1 *When supplying a resistive load such as an incandescent lamp the power in watts expended by an ac circuit is the product of the rms value of the voltage and current.*

When *an ac circuit consists of a* ***load that is made up of resistance and inductance, such as a motor*** some of the current flowing in the circuit is required to supply the load and some of the current is required to build a magnetic field around an inductor coil. *A motor has coils of wire in the stator so it is an inductive load.* Energy is stored in this magnetic field at the load. If the circuit is de-energized and the voltage taken away the current supporting that magnetic field will stop and the magnetic field will collapse. That collapsing magnetic field will induce a voltage into the inductor wire. The energy that was stored in the magnetic field at the load is sent back to the source. This is called *reactive power*. For an ac circuit consisting of resistance and inductance or a circuit consisting of resistance and capacitance some power in the circuit will be real power that is expended and some will be reactive power that is actually returned to the source. The voltage and current in the circuit will be out of alignment by some angle theta (θ) and the product of the voltage and current (volt-amperes) will be a value greater than the actual real power in watts expended by the circuit.

In order to understand this discussion of power in ac circuits it is important to represent voltage, current, and impedance in polar and rectangular form. These quantities are explained in some detail in *Tech Note 221*. If the resistance (R), inductive reactance (X_L), and capacitive reactance (X_C) of a circuit are known and arranged in series the magnitude of the impedance (Z) of that circuit can be determined using *Equation 223.1.*

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \textbf{\textit{Eq. 223.1}}$$

Where reactance (X_L) is present in an ac circuit along with resistance (R) the current sine wave will be out of alignment with the voltage sine wave. The amount of this misalignment is expressed in degrees and will be some value between minus ninety degrees (-90°) and plus ninety degrees (+90°). To determine the angle of shift of the current from the voltage the equation is the inverse tangent of the reactance divided by the resistance (R) as shown in *Equation 223.2.* <u>*If the net reactance is positive the current will be lagging behind the voltage,*</u> and thus the angle will be negative. If the net reactance is negative current will be leading the voltage and the angle will be positive. The following example will shown how the magnitude of the impedance (Z_{RL}) and the shift angle are determined for an inductive circuit.

$$\theta = \tan^{-1}\frac{X}{R} \qquad \textbf{\textit{Eq. 223.2}}$$

Example: A circuit, shown in *Figure 223.2* is powered by a 120 volt, 60 Hz ac supply and consists of an 8.4 ohm resistor in series with an inductor with an inductive reactance (X_L) of 10 ohms. Determine the impedance of the circuit in polar form.

Answer: First determine the magnitude of the impedance using *Equation 223.1* and then the angle between the voltage and current can be determined using *Equation 223.2.*

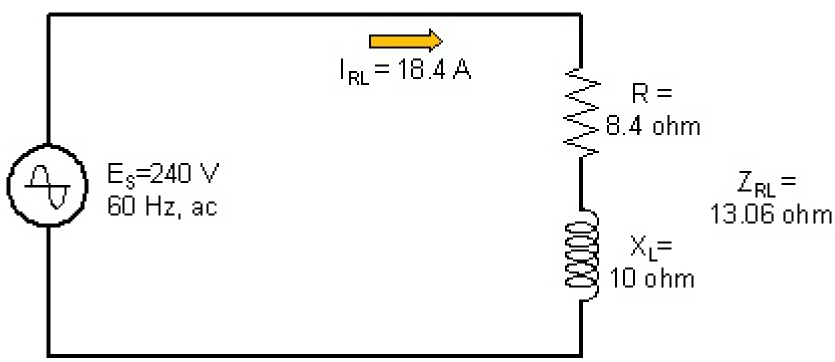

Figure 223.2 *A circuit powered by a 240 volt, 60 Hz, ac supply consists of a 8.4 ohm resistance in series with an inductive reactance of 10 ohms has an impedance of 13.06 ohm ∠+50°. The current is shown both in polar and rectangular form.*

$$Z = \sqrt{8.4^2 + 10^2} = 13.06\Omega$$

$$\theta = \tan^{-1}\frac{10}{8.4} = +50^{o}$$

Refer to *Tech Note 221* for a review of multiplication and division of quantities such as voltage, current, and impedance that are expressed in polar form. The impedance of the circuit shown in *Figure 223.2* was determined in polar form as Z = 13.06 ohm $\angle$+50°. The voltage of the circuit was given as 240 volts $\angle$0°. *Equation 223.3* is simply Ohm's law arranged to determine current when the voltage and impedance of the circuit are known.

$$I = \frac{E}{Z} \qquad \textbf{\textit{Eq. 223.3}}$$

Example continued: Next divide the circuit voltage by the impedance to determine the current using *Equation 223.3.* Divide the magnitude of the voltage by the magnitude of the impedance to obtain the magnitude of the current. To determine the angle the current is shifted from the voltage. Change the sign of the angle in the denominator (Z) and add it to the angle in the numerator. The current for the circuit of *Figure 223.2* will be 18.4 amperes with an angle of minus fifty degrees (-50°). Both the voltage and current of the circuit of *Figure 223.2* are represented in polar form and as sine waves in *Figure 223.3.* For this inductive circuit, the current is lagging behind the voltage by 50°.

$$I = \frac{240 \angle 0^\circ \text{ V}}{13.06 \angle +50^\circ\ \Omega} = 18.4 \angle -50^\circ \text{ A}$$

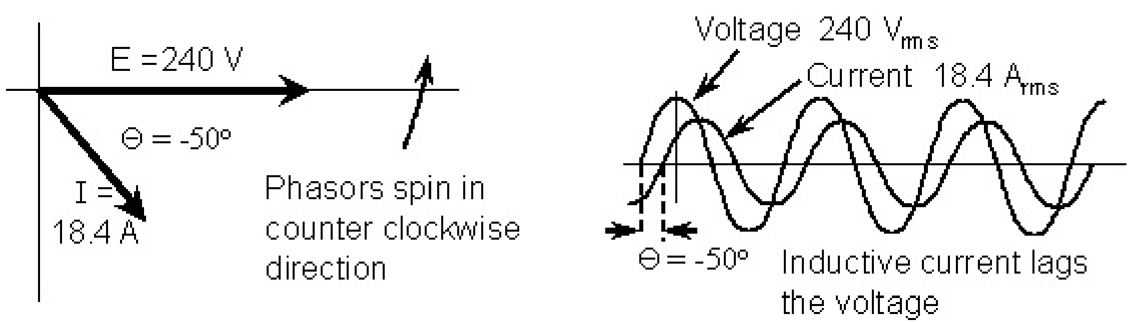

Figure 223.3 *The current for the inductive circuit of Figure 223.2 is **lagging** behind the voltage by an angle of 50°.*

Effect of Inductance & Capacitance on Circuit Current: When a load supplied by an ac circuit is pure resistance there is no significant magnetic or electrostatic field present to influence the relationship of circuit voltage and current. The current and voltage of the ac circuit will be sine waves that are in alignment with each other. The current and voltage will be zero at the same time and they will reach a maximum at the same time as shown in *Figure 223.4.* If the current and voltage sine waves are multiplied together another sine wave pattern will be created with all values positive. The value of power at any instant in time will be the point on the volt-ampere (apparent power) curve. The energy expended by the circuit will be the net positive area under the volt-ampere curve. In the case of a resistive load supplied by an ac source all of the area under the volt-ampere curve is positive. Note in *Figure 223.4* the current and voltage are in perfect alignment and the angle theta (θ) is zero. If the current and voltage were represented in polar form, such as in *Figure 223.3*, the angle θ would be zero and the current and voltage would point in the same direction.

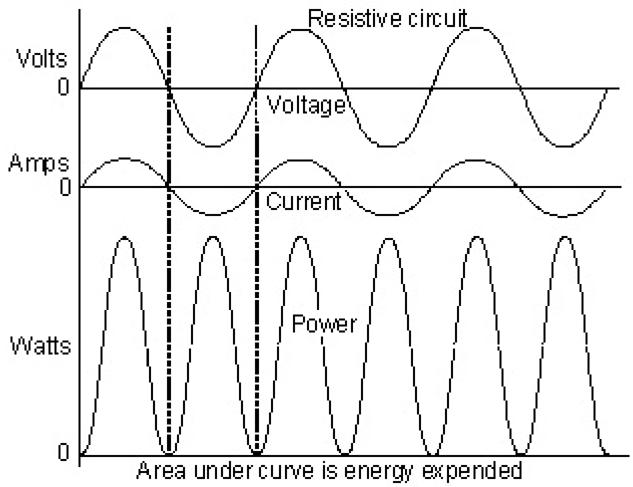

Figure 223.4 *In the case of an ac circuit with a purely resistive load the voltage and current will be in perfect alignment and the shift angle θ will be zero and the product of voltage and current will all be positive.*

When an ac circuit supplies a load that consists of inductance or capacitance in addition to resistance, the result will be a shift in the current sine wave with respect to the voltage sine wave by some angle θ. *Figure 223.5* represents the voltage and current in an ac circuit with an inductive load where the current is lagging behind the voltage by an angle θ of 50°. This is the same off-set angle as in the previous example. Note in *Figure 223.5* that when the voltage and current are not in perfect alignment there will be times when the voltage and current have opposite signs thus resulting in negative volt-amperes. The real power expended in the circuit is the net positive volt-amperes. When the voltage and current sine waves are out-of-alignment the apparent power is greater than the real power. Real energy expended in *Figure 223.5* is the net positive area between the volt-ampere curve and the zero line.

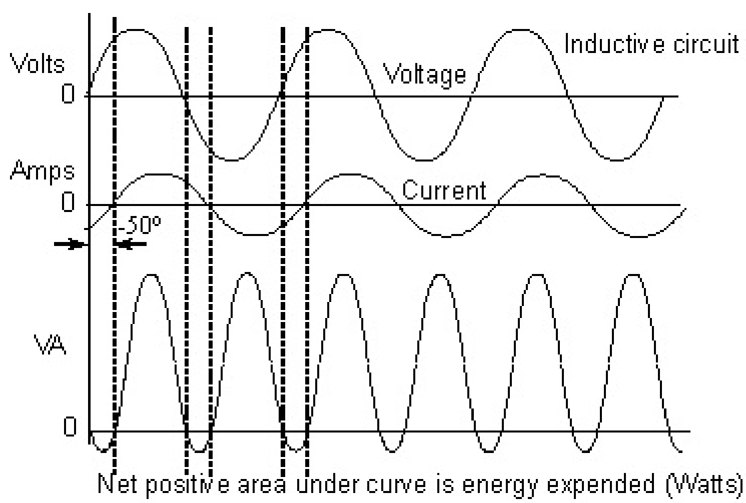

Figure 223.5 *In the case of an ac circuit where the load consists of inductance as well as resistance the current will lag behind the voltage by an angle θ and some of the volt-amperes will be negative. Real power expended in the circuit is the net positive volt-amperes.*

Apparent power: The apparent power of an ac circuit is in volt-amperes and is simply the product of the voltage and current. The apparent power forms the hypotenuse of a right triangle where the real power is the horizontal leg and reactive power is the vertical leg. *Figure 223.6* is a diagram showing the relationship of real power, apparent power, and reactive power. Real power is the quantity that is expended by the circuit. It is energy that has been used doing work and producing heat or light. Once used this energy is gone. In an inductor energy is stored in a magnetic field and that energy is returned to the circuit when the current that produces it stops. Likewise in a capacitor energy is stored in the dielectric in the form of an electric field. That energy is returned to the circuit when the voltage that produces it stops and the capacitor is discharged. This energy that is stored by an *inductor or a capacitor* is called *reactive power*. This reactive power requires current in the circuit but it does no work. In *Figure 223.6* the reactive power is represented by the vertical component of the power diagram. The *apparent power* is the hypotenuse of the right triangle created by the real power on the horizontal side and the reactive power on the vertical side. The apparent power is the combined effect of the real power and reactive power. *Apparent power is simply the product of voltage and circuit current* shown in *Equation 223.4.* The *units of apparent power are volt-amperes (VA).* The values of apparent power, real power, and reactive power for the previous example are also shown in *Figure 223.6.* How these values are determined is explained in the following discussion.

Apparent Power = Voltage X Current **_Eq. 223.4_**

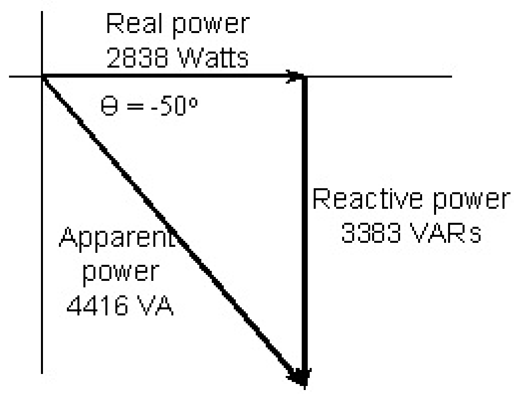

Figure 223.6 *Apparent power is the product of voltage and current in an ac circuit and it forms the hypotenuse of a right triangle on a power diagram with an angle θ with respect to the horizontal axis. The horizontal leg of the triangle is the real power expended in the circuit and the vertical leg is the reactive power associated with the magnetic or electrostatic field of the load.*

Since the power diagram of *Figure 223.6* is a right triangle, trigonometry can by used to determine the real power and the reactive power of the circuit given the apparent power and the angle (θ). *The real power is the product of voltage and current (VA) times the cosine of the angle θ* as shown in *Equation 223.5.* The ***reactive power*** is the product of voltage and current times the sine of the angle θ as shown in *Equation 225.6.* ***Real power*** *is in units of watts and reactive power is in units of VARs (volt-amperes reactive).*

Real Power = VA × Cos θ **_Eq. 223.5_**

Reactive Power (VAR) = VA × Sin θ **_Eq. 223.6_**

Example continued: Determine the apparent power, real power, and reactive power for the example of *Figure 223.2.* To determine apparent power use *Equation 223.4.* Real power is determined using *Equation 223.5*, and reactive power is determined using *Equation 223.6.* These values are displayed in *Figure 223.6.*

Apparent Power = $240\ \angle 0°$ V × $18.4\ \angle -50°$ A = $4{,}416\ \angle -50°$ VA

Real Power = 4,416 VA × Cos (−50°) = 2,838 watts

Reactive Power = 4,416 VA × Sin (−50°) = 3,383 VARs

Power Equation: The commonly known equation for determining power in an ac circuit is shown in *Equation 223.7.* This is for a single-phase circuit. *Power is the product of the voltage, current, and the power factor.* This equation is derived from *Figure 223.6,* and *power factor is the cosine of the angle θ*. *Power factor is 1.0* when the angle is zero. Circuits discussed thus far are single-phase circuits. This means the ac source produces only one sine wave to power the circuit. Frequently circuits for which power must be determined are 3-phase circuits where there are three conductors from the source with three different sine wave combinations between the pairs of conductors. Each of these sine waves are spaced 120° off-set from each other (See *Tech Note 220*). Typically the voltage between the three conductors will be nearly identical. The current for a specific 3-phase load will be approximately the same in each conductor. For such a load one current and one voltage are used in the power equation except for 3-phase power the equation has the *voltage, current, and power factor multiplied by the square root of three* which is the number 1.73. To determine power for a 3-phase circuit use *Equation 223.8.*

Single-Phase Power:

Power = Voltage × Current × power factor ***Eq. 223.7***

3-Phase Power:

Power = 1.73 × Voltage × Current × power factor ***Eq. 223.8***

Power Factor: Examining the previous example and *Figure 223.6* and it can be seen that the real power of an ac circuit is the voltage times the current times the cosine of the angle θ. For *an ac circuit where the load is pure resistance the angle θ will be 0° and Cos 0° is 1.0.* For the opposite extreme the angle θ may be either +90° (capacitive) or it may be −90° (inductive). In either case Cos +90° or Cos −90° is zero. This means no matter how much current is flowing, there is no real power being expended by the circuit. The energy stored in the magnetic field or electrostatic field of the load is sent back to the source when the voltage is removed. The values of Cos θ then varies from 0.0 to 1.0 depending upon the ratio of resistance to reactance in the circuit. *Cos θ in the power equation is called the power factor.* Power factor can be measured directly or it can be determined by measuring the circuit voltage and current and measuring the power draw of the circuit. Power factor is calculated from *Equation 223.7* by dividing the wattage by the volt-amperes.

Single-Phase Circuit Power Factor

$$\text{power factor} = \frac{\text{Watts}}{\text{Volt} \times \text{Amperes}} \qquad \textit{Eq. 223.9}$$

3-Phase Circuit Power Factor

$$\text{power factor} = \frac{\text{Watts}}{1.73 \times \text{Volts} \times \text{Amperes}} \qquad \textit{Eq. 223.10}$$

Significance of Power Factor: An ac circuit that operates with a small shift between the current and voltage sine waves has an apparent power with a lower magnitude than a circuit with a large angle θ. This is illustrated in *Figure 223.7* for a load with a large and a small shift between the voltage and the current. Since the circuit voltage remains constant the load with the greater angle θ will have a higher current for the same real power output. Electrical system conductors and equipment must be sized to carry the current of the load so it is important to minimize the current required to supply a particular load by keeping the power factor high (small angle) whenever practical. The current drawn by a circuit can be determined by dividing the power in watts by the circuit voltage and power factor as shown in *Equation 223.11.* This equation is for a single-phase load. For a 3-phase load it is necessary to also divide by **1.73**. The inductive reactance of common ac loads such as induction motors can be offset by the *installation of a capacitor connected in parallel with the motor*. The capacitor is installed at the motor which then changes the angle between the voltage sine wave and the circuit current sine wave. By reducing this angle the power factor is increased and the circuit current drawn by the motor is reduced as shown in *Figure 223.7.*

$$\text{Current} = \frac{\text{Power in watts}}{\text{Voltage} \times \text{power factor}} \qquad \textit{Eq. 223.11}$$

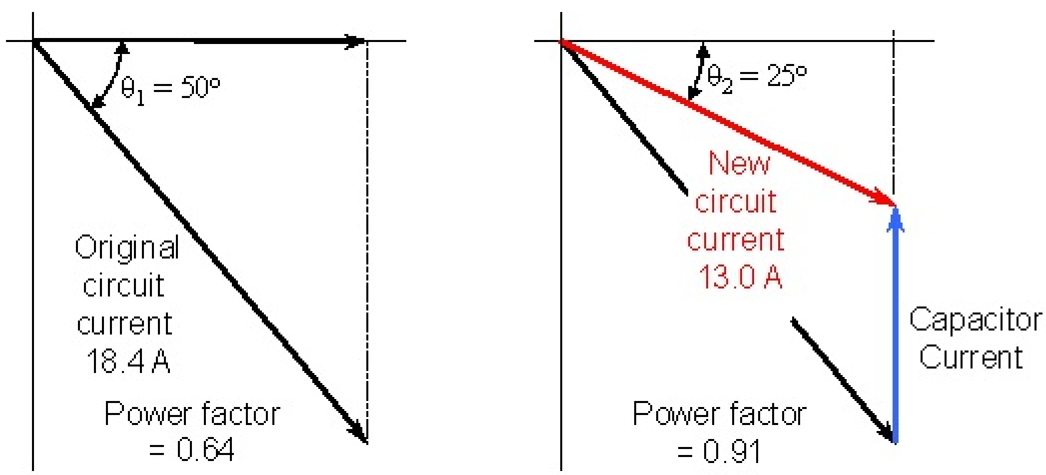

Figure 223.7 *The diagram on the left shows the original circuit current with a low power factor. By reducing the power factor as shown with the diagram on the right the circuit current is reduced by increasing the power factor.*

If the current sine wave is lagging the voltage sine wave the circuit is considered to be inductive. The power factor is the cosine of the angle the current is lagging the voltage.

Power factor = Cos θ ***Eq. 223.12***

θ = Cos^{-1} (power factor) ***Eq. 223.13***

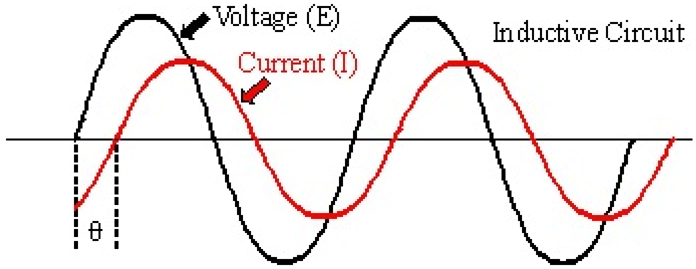

Figure 223.8 *Inductance in an ac circuit caused by a device such as an electric motor will cause the current sine wave to lag behind the voltage sine wave.*

Conclusion: For a pure ***resistive circuit*** the voltage sine wave and the current sine wave are in perfect alignment. They cross the zero line at the same time and reach their peak values at the same time. A vector diagram is shown to the right with the vectors spinning counter-clockwise at 60 revolutions per second for ac power at 60 Hz.

For an ***inductive circuit*** notice the voltage and current sine waves are not in alignment. The *current sine wave is lagging behind the voltage sine wave* at some angle which can be as much as nearly 90° depending upon how much inductance is in the circuit.

For a ***capacitive circuit*** the *current sine wave leads the voltage sine wave* and they are not in alignment. The current sine wave can lead the voltage sine wave as much as nearly 90° depending upon the amount of capacitance in the ac circuit.