Electrical Tech Note — 205

Biosystems & Agricultural Engineering Department
Michigan State University

Alternating Current

Alternating current is produced commercially by spinning a magnetic field inside of a set of coils of wire. When there is motion between a wire and a magnetic flux such that the wire is cutting across perpendicular to the orientation of the magnetic flux a voltage is induced into the wire. Since the magnetic field has two poles (North and South) spinning the magnetic field inside stationary coils will create current flow in the stationary coils. The polarity of the voltage induced into the coil of wire will reverse each half revolution of the magnetic field. Over time the voltage will build up to a maximum then decrease to zero completing half a revolution of the magnetic filed. Then the polarity will reverse and the voltage will again build up to a maximum then decrease to zero. Viewed over time the voltage will trace out a sine wave each revolution of the magnetic flux. The frequency of the sine wave will be the revolutions per second of the magnetic flux inside the generator. Commercially available electrical power in the United States has a frequency of 60 Hz which means the generator magnetic field is spinning at a rate of 3,600 revolutions per minute, 60 revolutions per second. The purpose of this *Tech Note* is to review the basic form in which alternating current is represented mathematically as a sine wave function.

Radians, Degrees, & Angular Frequency: The circumference of a circle is πd or $2\pi r$. If r = 1 then the circumference of a circle is πd. The orientation of the vector "r" can be specified at any time in terms of the distance traversed around a circle such as $\pi/4$ which is +45°. Rather than specifying the position of a vector about a circle which in degrees, position can be specified in terms of radians where a complete circle is 2π radians. One radian is actually 57.3°. This concept is summarized in *Figure 205.1.*

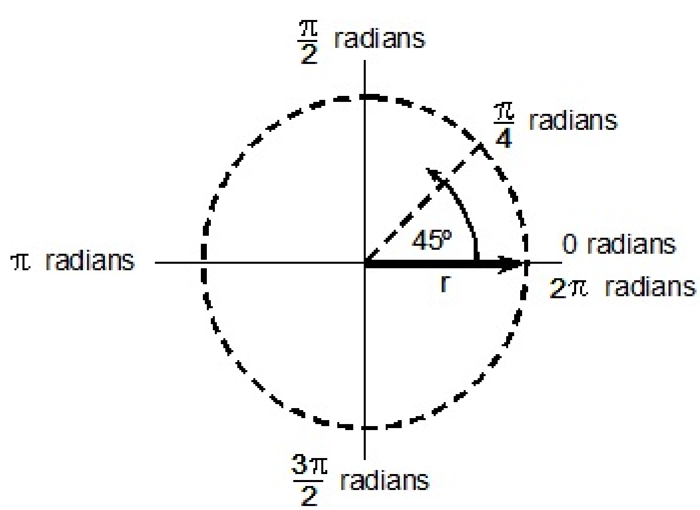

Figure 205.1 *Position of a vector (radius) about a circle can be specified in degrees or in radians where a complete circle is 2π radians.*

A sine wave can be generated by viewing the position of the tip of a vector representing the radius of a circle as it spins at a constant speed over a period of time. Refer to *Figure 205.1* and imagine the center of the circle moving to the right as time passes while the vector is spinning in a counter-clockwise direction. A sine wave will be generated as shown in *Figure 205.2.* The rate at which the vector is spinning is the angular frequency omega (ω) which is generally specified in radians per second. Frequency of a sine wave is actually equivalent to the revolutions per second the vector is spinning to generate the sine wave. Since a complete circle is 2π radians, dividing the angular frequency (ω) by 2π radians will give the frequency (f) in cycles per second or Hz. Angular velocity (ω) is equal to 2π f. It is customary to represent the sine wave in the form of a cosine function as shown in *Figure 205.2* where an initial angle in radians is introduced to shift the cosine function so that it starts at zero time at a value of zero.

$$\omega = 2 \times \pi \times f = 6.28 \times f \qquad \textbf{Eq. 205.1}$$

$$f = \frac{\omega}{2 \times \pi} = \frac{\omega}{6.28} \qquad \textbf{Eq. 205.2}$$

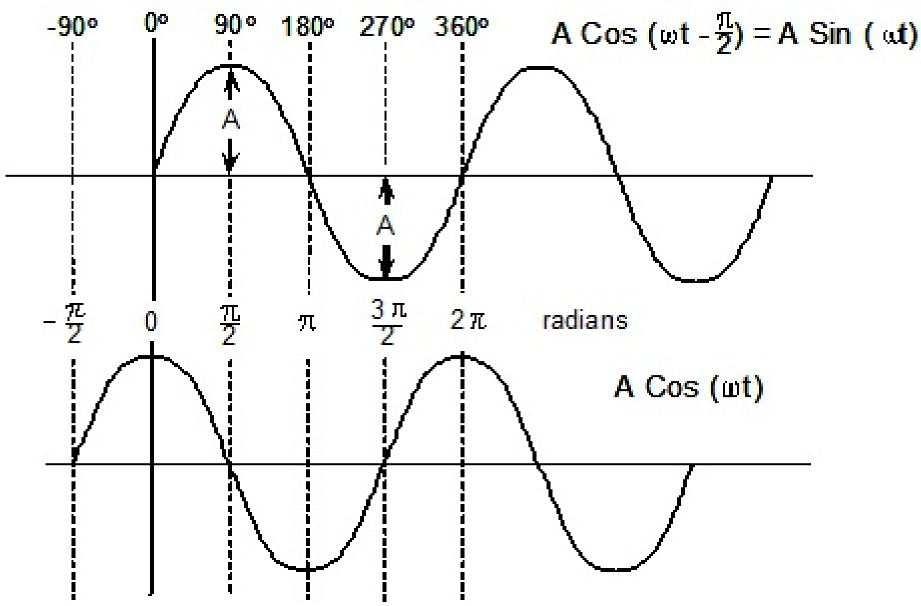

Figure 205.2 *A sine wave can be represented with a cosine function by introducing a phase shift or angle shift so that the function at time zero starts at zero rather than the value one.*

Effective or rms Value of a Sine Wave: Assume a voltage sine wave can be described with the function v(t) = 170 Cos (377t - 1.57). The value 170 is the positive and negative peak (E_{PEAK}) of the voltage wave. But this is not the steady state value of the voltage that can be used to determine desired steady state quantities of the circuit. The value that would be used in an Ohm's law calculation or a power calculation is called the ***"effective"*** value or ***"root mean square"*** **(rms)** value of the voltage ($E_{effectiive}$) or (E_{rms}). For a perfect sine wave or cosine wave the ratio of the *peak value* to the *effective value* is the square root of two (1.414). If the effective value of the function v(t) = 170 Cos (377t - 1.57) is desired, divide 170 by 1.414 will result in an effective (rms) value of 120 volts. This is illustrated in *Figure 205.3.*

$$E_{effective} = \frac{E_{peak}}{1.414}$$ **Eq. 205.3**

$$E_{peak} = E_{effective} \times 1.414$$ **Eq. 205.4**

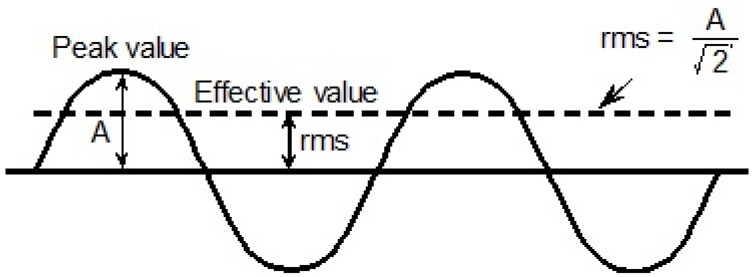

Figure 205.3 *Divide the peak value by 1.414 to get the effective (rms) value of the sine wave. (Rms is root mean square)*

Setting Initial Starting Point for Cosine Function: For this discussion the voltage function v(t) = 170 Cos (377t - 1.57) will be used. A cosine function at *time zero* and *angle zero* will have a value of 1 cycle. The function will appear like the top function in *Figure 205.4.* In order for the cosine wave to have a value of zero at time zero and appear as though it is a sine wave set function starting point at -90° which is -1.57 radians. Compare the two cosine functions in *Figure 205.4* with and without the initial starting point set at -1.57 radians.

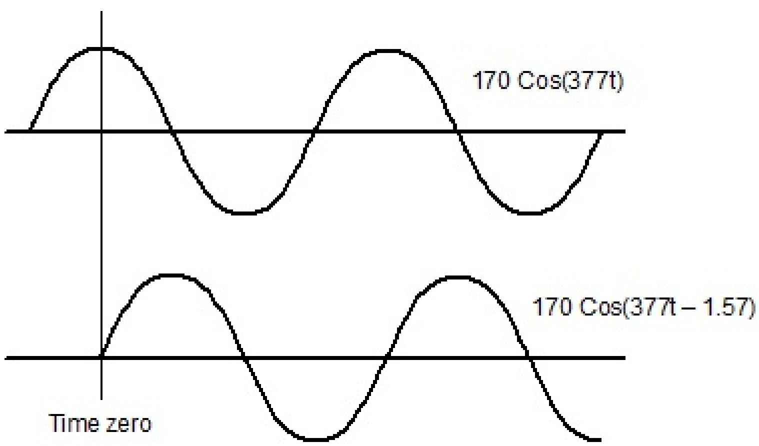

Figure 205.4 *Compare the trace of the cosine function with the starting point at 0 radians (top trace) with the bottom trace where the cosine function has a starting point at -1.57 radians.*

Period of a Sine Wave: ***Frequency (f)*** is the number of sine wave cycles that occur in one second. **Period (T)** is the length of time required to complete one cycle. The unit of *Hertz* is 1/s. Therefore, the ***period*** is the reciprocal of frequency (*Eq. 205.5*). *Figure 205.5* illustrates the period of a sine wave. *(Lower case s represents seconds)*

$$T = \frac{1}{f}$$ **Eq. 205.5**

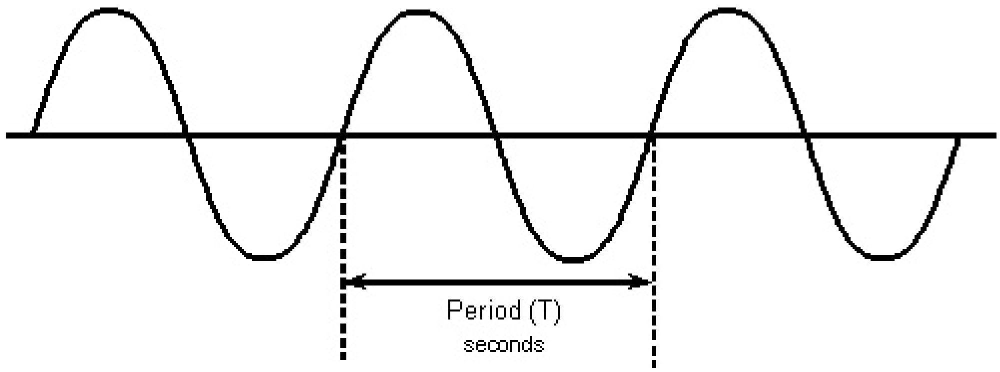

Figure 205.5 *The period of a sine wave is the time required to complete one cycle. If the frequency is 60 Hz the value of T is 0.0167 s. (At 50 Hz T is 0.025)*

At very high frequencies waves such as electromagnetic waves are described in a unit called wavelength *lambda* (λ), illustrated in *Figure 205.6.* In order to determine the wavelength of a sine wave the velocity (v) of the wave through a medium must be known. Wavelength is the distance of one cycle of a sine wave in meters (m). The distance one cycle a sine wave travels in one second. At 60 Hz the wavelength of one cycle is approximately 3,100 miles. The speed of light through air is considered to be 2.997 x 10^8 m/s. Wavelength (λ) of a sine wave is considered to be the velocity (v) of the wave divided by the frequency (f) in *Hertz (Eq. 205.6)*.

$$\lambda = \frac{v}{f} \qquad \textbf{Eq. 205.6}$$

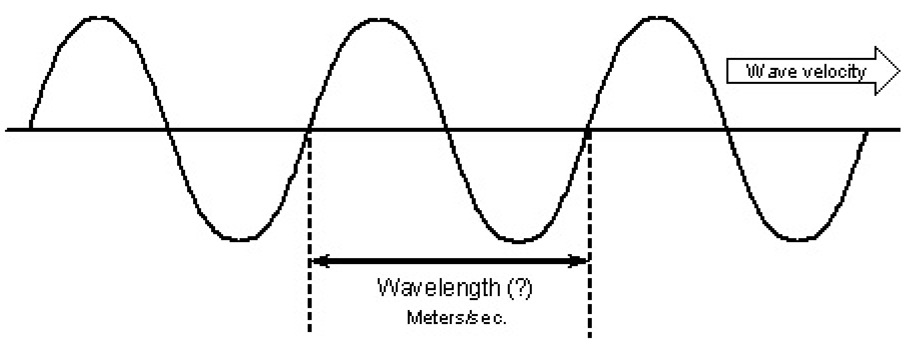

Figure 205.6 *Wavelength is the distance traveled by the sine wave in one second at the velocity of the wave. The distance traveled by one 60 Hz sine wave in one second is approximately 3,100 miles. (1 foot = 0.3048 m)*